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THEORY OF NOISY TWO-PORT NETWORKS *

BY

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ABSTRACT

A new geometric-analytic theory of noisy two-port networks is presented. It is based, geometrically, on the isometric sphere method, a generalization of the isometric circle method to three dimensions, and, analytically, on a three-dimensional conformal transformation which was originally derived by Poincaré and Picard. The transformation is used in a study of transformations of noise ensemble average ratios through bilateral two-port networks. The study is facilitated by interpreting the transformations as non-Euclidean movements in the Poincaré and Cayley-Klein models of three-dimensional hyperbolic space. By a simple extension, noise ensemble average transformations are performed in four-dimensional spaces by means of four-vectors which are analogous to the Stokes vectors used in optics and antenna theory.

The following problems are studied by means of the new theory: the Rothe and Dahlke method of splitting a noisy two-port network into noisy and noise-free parts, cascading of noisy two-port networks, noise tuning and noise matching, the wave representation of noisy two-port networks, and the optimum noise factor.

INTRODUCTION

The rapid development of transistor amplifiers and longitudinal electron beam amplifiers has led to the creation of linear noise theories. This paper presents a new linear noise theory. The theory is based on a conformal transformation method called "the isometric sphere method." The method is a straightforward extension of the isometric circle method to three dimensions. A further extension of the isometric sphere

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method introduces a four-vector which is analogous to the well known Stokes vector used in optics and antenna theory.

Analytically the theory of noisy two-port networks consists of a direct generalization of transformation equations for noise-free two-port networks. It has the advantage of yielding immediate geometric interpretations of noise transformations as transformations in three-and four-dimensional linear spaces.

In a recent paper in this JOURNAL (1)[2] and in two recent notes (2, 3) it has been outlined how an impedance transformation through a bilateral two-port network can be geometrically represented by a non-Euclidean movement in a Poincaré or a Cayley-Klein model of three-dimensional hyperbolic space. By using the Cayley-Klein model points on the surface of the unit sphere, corresponding to impedance quantities, are transformed into points also on the surface of the unit sphere. However, points inside the surface of the unit sphere are simultaneously transformed into points inside the surface. Now a natural question is: "What physical interpretation can we give to these points *inside* the surface of the unit sphere?" The answer is that the points may be thought of as representing noise ensemble average ratios. Thus, transformations of noise ensemble average ratios through noise-free bilateral two-port networks can be geometrically represented by non-Euclidean transformations in models of three-dimensional hyperbolic space. The special cases of impedance transformations through noise-free bilateral two-port networks are obtained as non-Euclidean transformations of points on the absolute surfaces of the different models.

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THE ISOMETRIC SPHERE METHOD

The Isometric Circle Method

The input voltage V' and the input current I' of a noise-free bilateral two-port network are linearly related to the output voltage V and the output current I:

$$\psi' = \begin{pmatrix} V' \\ I' \end{pmatrix} = \begin{pmatrix} ab \\ cd \end{pmatrix} \begin{pmatrix} V \\ I \end{pmatrix} = T\psi; \quad ad - bc = 1. \tag{1}$$

The input impedance $Z' = V'/I'$ is expressed in terms of the output impedance $Z = V/I$ by the linear fractional transformation:

$$Z' = \frac{aZ + b}{cZ + d}. \tag{2}$$

The linear fractional transformation, which is sometimes called a homographic or a Möbius transformation, is the most general one-to-one and directly conformal transformation that transforms the entire complex

[2] The boldface numbers in parentheses refer to the references appended to this paper.

plane into itself. The conformal property suggests the use of graphical methods. The isometric circle method (4, 2) is a method of this kind.

The isometric circle is defined as the circle that is the complete locus of points in the neighborhood of which lengths are unaltered in magnitude by the linear fractional transformation. The isometric circle of the direct transformation, C_d, has its center at $O_d = -d/c$ and radius $R_e = 1/|c|$; the isometric circle of the inverse transformation C_i, has its center at $O_i = a/c$ and the same radius (see Fig. 1). The isometric

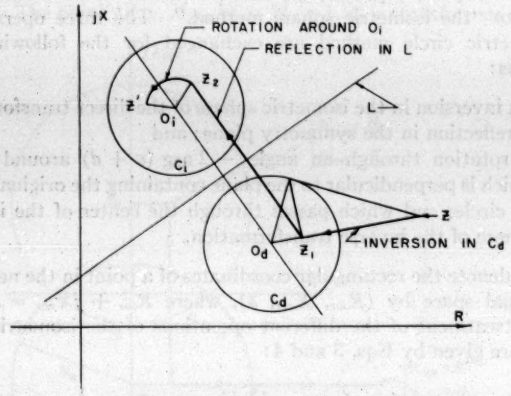

FIG. 1. The isometric circle method.

circle method is composed of the following constructions:

1. An inversion in the isometric circle of the direct transformation C_d, $Z \rightarrow Z_1$;
2. A reflection in the symmetry line L to the two circles, $Z_1 \rightarrow Z_2$; and
3. A rotation around the center O_i of the isometric circle of the inverse transformation C_i through an angle $-2 \arg (a + d)$, $Z_2 \rightarrow Z'$.

For lossless two-port networks $a + d$ is real and the third operation is eliminated and a "non-loxodromic" transformation is obtained. For lossy two-port networks, yielding "loxodromic" transformations, the third operation must be considered. The simplicity of the use of the isometric circle method in the complex plane is seriously hampered by the third operation, the rotation. It has been shown (5), however, that if the complex plane is stereographically mapped on the surface

of the unit sphere, which is again considered to be the absolute surface of the Cayley Klein model of three-dimensional hyperbolic space, then the linear fractional transformation (2) can be performed by merely two non-Euclidean reflections in that space.

The Isometric Sphere Method

If the isometric circles are replaced by (hemi-)spheres and the symmetry lines of the isometric circles is replaced by the symmetry plane of the (hemi-)spheres, the isometric circle method can be generalized to "the isometric sphere method." The three operations of the isometric circle method are exchanged for the following three operations:

1. An inversion in the isometric sphere of the direct transformation;
2. A reflection in the symmetry plane; and
3. A rotation through an angle $-2\arg(a+d)$ around an axis which is perpendicular to the plane containing the original isometric circles and which passes through the center of the isometric sphere of the inverse transformation.

If we denote the rectangular coordinates of a point in the new three-dimensional space by (R_{eor}, X_{eor}, h), where $R_{eor} + jX_{eor} = Z_{eor}$, the analytic treatment of the different operations of the isometric sphere method are given by Eqs. 3 and 4:

$$\left.\begin{aligned} Z_{eor_1} + \frac{d}{c} &= \left(Z_{eor} + \frac{d}{c}\right)\frac{h'}{h} \\ Z_{eor_2} &= \frac{-\dfrac{a+d}{c}Z^*_{eor_1} + \dfrac{aa^* - dd^*}{cc^*}}{\dfrac{a^* + d^*}{c^*}} \\ Z'_{eor} - \frac{a}{c} &= \left(Z_{eor_2} - \frac{a}{c}\right)e^{-j^2\arg(a+d)} \end{aligned}\right\} \tag{3}$$

$$h' = \frac{h}{cc^*s^2 + cd^*Z_{eor} + dc^*Z^*_{eor} + dd^*}. \tag{4}$$

If we combine Eqs. 3 and 4 and put

$$s^2 = Z_{eor}Z^*_{eor} + h^2 \tag{5}$$

the isometric sphere method is expressed in analytic form by the follow-

ing equations:

$$\left.\begin{aligned} s'^2 &= \frac{aa^*s^2 + ab^*Z_{cor} + ba^*Z^*_{cor} + bb^*}{cc^*s^2 + cd^*Z_{cor} + dc^*Z^*_{cor} + dd^*} \\ Z'_{cor} &= \frac{ac^*s^2 + ad^*Z_{cor} + bc^*Z^*_{cor} + bd^*}{cc^*s^2 + cd^*Z_{cor} + dc^*Z^*_{cor} + dd^*} \end{aligned}\right\} \quad (6)$$

Example of the Use of the Isometric Sphere Method

A simple example of the use of the isometric sphere method is shown in Fig. 2. The point $(R_{cor}, X_{cor}, h) = (4, -1, 3)$ is transformed through

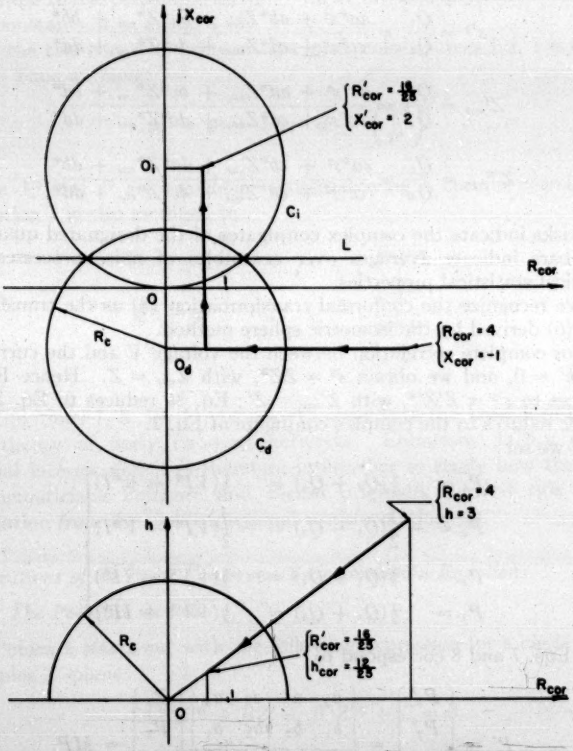

FIG. 2. Example of the use of the isometric sphere method.

a lossless bilateral two-port network $(a = 1,\ b = -j,\ c = -j/2,\ d = 1/2)$ to $(R'_{cor}, X'_{cor}, h') = (16/25, 2, 12/25)$.

ANALYTIC REPRESENTATIONS OF NOISY TWO-PORT NETWORKS

A study of the isometric sphere method reveals that for a noise process, transformed by a noise-free two-port network, Eqs. 1 and 2 are exchanged for the following equations:

$$Q' = \begin{bmatrix} Q_1' \\ Q_2' \\ Q_3' \\ Q_4' \end{bmatrix} = \begin{bmatrix} \overline{V'V'^*} \\ \overline{V'I'^*} \\ \overline{V'^*I'} \\ \overline{I'I'^*} \end{bmatrix} = \begin{bmatrix} aa^* & ab^* & ba^* & bb^* \\ ac^* & ad^* & bc^* & bd^* \\ ca^* & cb^* & da^* & db^* \\ cc^* & cd^* & dc^* & dd^* \end{bmatrix} \begin{bmatrix} \overline{VV^*} \\ \overline{VI^*} \\ \overline{V^*I} \\ \overline{II^*} \end{bmatrix} = LQ \qquad (7)$$

$$s'^2 = \frac{Q_1'}{Q_4'} = \frac{aa^*s^2 + ab^*Z_{eor} + ba^*Z^*_{eor} + bb^*}{cc^*s^2 + cd^*Z_{eor} + dc^*Z^*_{eor} + dd^*} \qquad (8a)$$

$$Z'_{eor} = \frac{Q_2'}{Q_4'} = \frac{ac^*s^2 + ad^*Z_{eor} + bc^*Z^*_{eor} + bd^*}{cc^*s^2 + cd^*Z_{eor} + dc^*Z^*_{eor} + dd^*} \qquad (8b)$$

$$Z'^*_{eor} = \frac{Q_3'}{Q_4'} = \frac{ca^*s^2 + cb^*Z_{eor} + da^*Z^*_{eor} + db^*}{cc^*s^2 + cd^*Z_{eor} + dc^*Z^*_{eor} + dd^*}. \qquad (8c)$$

Asterisks indicate the complex conjugates of the designated quantities and bars indicate averages over ensembles of noise processes with identical statistical properties.

We recognize the conformal transformation (8) as the transformation (6) derived by the isometric sphere method.

For complete correlation between the voltage V and the current I, $h = h' = 0$, and we obtain $s^2 = ZZ^*$, with $Z_{eor} = Z$. Hence Eq. 8a reduces to $s'^2 = Z'Z'^*$, with $Z'_{eor} = Z'$; Eq. 8b reduces to Eq. 2, and Eq. 8c reduces to the complex conjugate of Eq. 2.

If we set:

$$\left.\begin{aligned} P_1 &= \tfrac{1}{2}(Q_2 + Q_3) = \tfrac{1}{2}(\overline{VI^*} + \overline{V^*I}) \\ P_2 &= -\frac{j}{2}(Q_2 - Q_3) = -\frac{j}{2}(\overline{VI^*} - \overline{V^*I}) \\ P_3 &= \tfrac{1}{2}(Q_1 - Q_4) = \tfrac{1}{2}(\overline{VV^*} - \overline{II^*}) \\ P_4 &= \tfrac{1}{2}(Q_1 + Q_4) = \tfrac{1}{2}(\overline{VV^*} + \overline{II^*}) \end{aligned}\right\} \qquad (9)$$

then Eqs. 7 and 8 correspond to:

$$P' = \begin{bmatrix} P_1' \\ P_2' \\ P_3' \\ P_4' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = MP \qquad (10)$$

and

$$\left.\begin{aligned} x' &= \frac{P_1'}{P_4'} = \frac{a_1x + a_2y + a_3z + a_4}{d_1x + d_2y + d_3z + d_4} \\ y' &= \frac{P_2'}{P_4'} = \frac{b_1x + b_2y + b_3z + b_4}{d_1x + d_2y + d_3z + d_4} \\ z' &= \frac{P_3'}{P_4'} = \frac{c_1x + c_2y + c_3z + c_4}{d_1x + d_2y + d_3z + d_4} \end{aligned}\right\} \tag{11}$$

where the 16 real constants, $a_1, a_2, \cdots d_3, d_4$, are all expressed in the complex constants, a, b, c, and d (5).

If the two-port network contains inner noise sources Eq. 1 is modified. Thus we have:

$$\psi'' = \psi^0 + T\psi; \quad \psi^0 = \begin{pmatrix} V^0 \\ I^0 \end{pmatrix} \tag{12}$$

where V^0 and I^0 are two noise sources having a certain correlation. Equation 7 is also modified to:

$$Q'' = Q^0 + LQ; \quad Q = \begin{bmatrix} \overline{V^0V^{0*}} \\ \overline{V^0I^{0*}} \\ \overline{V^{0*}I^0} \\ \overline{I^0I^{0*}} \end{bmatrix}. \tag{13}$$

Equations 7, 8, 10, 11, 12, and 13 constitute the basic formulas of the theory of noisy two-port networks. Equations 8*a*, *b*, *c* are of special interest and it is therefore interesting to study how the great mathematicians Poincaré and Picard originally derived this transformation from the linear fractional transformation (2).

Derivations of the Basic Conformal Transformation Equations

1. The Poincaré method

Poincaré starts out with the following expression for a circle in the complex Z'-plane:

$$AZ'Z'^* + BZ' + B^*Z'^* + C = 0 \tag{14}$$

where A and C are real constants and B is a complex constant. (The circle is denoted by C' in Fig. 3.) He then transforms the circle by the

linear fractional transformation (2) into another circle C:

$$\begin{aligned} & ZZ^*(A\ aa^* + B\ ac^* + B^*ca^* + C\ cc^*) \\ + & \ Z\ (A\ ab^* + B\ ad^* + B^*cb^* + C\ cd^*) \\ + & \ Z^*(A\ ba^* + B\ bc^* + B^*da^* + C\ dc^*) \\ + & \ 1\ (A\ bb^* + B\ bd^* + B^*db^* + C\ dd^*) = 0. \end{aligned} \tag{15}$$

Poincaré then generalizes to three dimensions. By using our previous notations the Poincaré extension consists in replacing ZZ^* by s^2, and Z by Z_{eor}. Thus Eq. 15 transforms into

$$\begin{aligned} & s^2(A\ aa^* + B\ ac^* + B^*ca^* + C\ cc^*) \\ + & \ Z_{eor}\ (A\ ab^* + B\ ad^* + B^*cb^* + C\ cd^*) \\ + & \ Z^*_{eor}(A\ ba^* + B\ bc^* + B^*da^* + C\ dc^*) \\ + & \ 1\ (A\ bb^* + B\ bd^* + B^*db^* + C\ dd^*) = 0. \end{aligned} \tag{16}$$

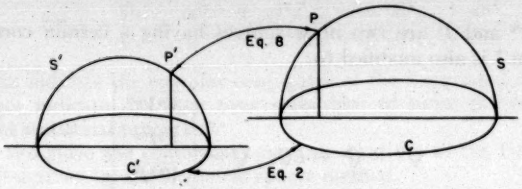

FIG. 3. The Poincaré method.

Equation 16 represents analytically the hemisphere S erected on the circle C (see Fig. 3). A point P on S has the coordinates (R_{eor}, X_{eor}, h). Similarly, a point P' having the coordinates (R'_{eor}, X'_{eor}, h') is situated on the hemisphere S' on C'. The equation for the latter hemisphere is:

$$As'^2 + BZ'_{eor} + B^*Z'^*_{eor} + C = 0. \tag{17}$$

If the constants A, B, and C are eliminated between Eqs. 16 and 17 the conformal transformation (8) is obtained.

Poincaré (6) showed that the transformation (8) can be considered to be a non-Euclidean transformation in a model of three-dimensional hyperbolic space having the Z_{eor}-plane as the absolute surface. The model is nowadays usually called "the Poincaré model" (1). Poincaré used the model, for example, in studying automorphic functions.

2. The Picard method

The second method of deriving the conformal transformation (8) is due to Picard (7, 8). Picard starts out with a linear fractional trans-

formation

$$w' = \frac{aw + b}{cw + d'} \tag{18}$$

where $w = u + jv$, $w' = u' + jv'$, and a, b, c, and d are real constants. He then calculates the square of the absolute value of w' and the real part u' of w':

$$\left.\begin{aligned} u'^2 + v'^2 &= \frac{a^2(u^2 + v^2) + 2abu + b^2}{c^2(u^2 + v^2) + 2cdu + d^2} \\ u' &= \frac{ac(u^2 + v^2) + (ad + bc)u + bd}{c^2(u^2 + v^2) + 2cdu + d^2} \end{aligned}\right\}. \tag{19}$$

We recognize that if we consider $Z_{eor} = R_{eor}$, and a, b, c, and d real in Eq. 8, then Eq. 8 reduces to the form of Eq. 19. Actually, Picard obtained Eq. 8 from Eq. 19 by studying the transformation of Hermitian forms by linear fractional transformations, and by putting $uu^* + v^2 = s^2$, $u = Z_{eor}$, and $v = h$.

Obviously Picard starts out in a plane perpendicular to the Z_{eor}-plane and cutting this plane along the R_{eor}-axis. He then extends the transformation into three dimensions. In modern technical language we may say, that while Poincaré extends a study of noise-free lossy two-port networks to a study of noisy lossy networks, Picard extends a study of noisy resistive networks (with real correlation between the voltage and the current) to a study of noisy lossy two-port networks.

Thus we find that the extension of the isometric circle method to the isometric sphere method, which constitutes the basis of the new theory of noisy two-port networks, is an extension in the sense of Poincaré's method.

The conformal transformation (8) has been extensively used by Fricke and Klein (9) in a study of automorphic functions and it has also been derived in the textbook by Hurwitz and Courant (10).

GEOMETRIC REPRESENTATIONS

Connections Between the Poincaré and Cayley-Klein Models of Three-Dimensional Hyperbolic Space

Before we apply the basic transformations (8) and (11) in the theory of noisy two-port networks, let us look into their geometric representations.

We know (from Poincaré) that Eq. 8 may be considered to be a non-Euclidean transformation in a three-dimensional hyperbolic space having the Z_{eor}-plane as the absolute surface. This model (turned up-side-down and in double scale) is transformed into a unit sphere by a simple inversion as shown in Fig. 4. A point Q in the figure, having the coordinates (R_{eor}, X_{eor}, h) is transformed into a point P_0, having the

coordinates (x_0, y_0, z_0), by the equations:

$$\left.\begin{aligned} x_0 &= \frac{2R_{cor}}{R^2_{cor} + X^2_{cor} + (h+1)^2} \\ y_0 &= \frac{2X_{cor}}{R^2_{cor} + X^2_{cor} + (h+1)^2} \\ 1 - z_0 &= \frac{2(h+1)}{R^2_{cor} + X^2_{cor} + (h+1)^2} \end{aligned}\right\}. \tag{20}$$

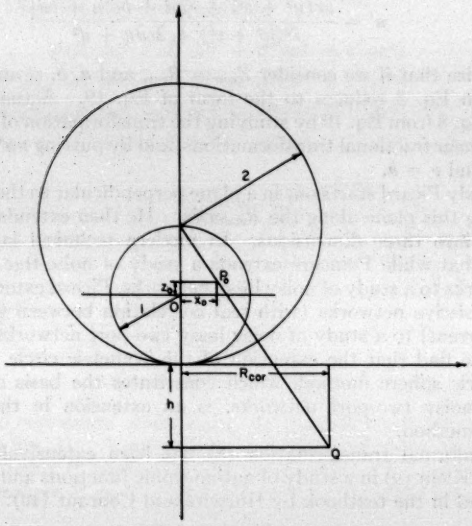

FIG. 4. Geometric connection between the Poincaré models of three-dimensional hyperbolic space.

A semicircle perpendicular to the Z_{cor}-plane is transformed into an arc of a circle orthogonal to the surface of the unit sphere. Thus the Poincaré model of three-dimensional hyperbolic space having the unit sphere as the absolute surface is obtained. We now transform this model into a Cayley-Klein model of three-dimensional hyperbolic space (also having the unit sphere as the absolute surface) by the "Darboux transformation" through which every pair of points which is inverse with respect to the sphere is transformed into the pole of its symmetry plane (11). The transformation is equivalent to the transformation ℜ that was introduced by Deschamps (12) in the two-dimensional case. A point P_0 with the coordinates (x_0, y_0, z_0) is transformed into a point P

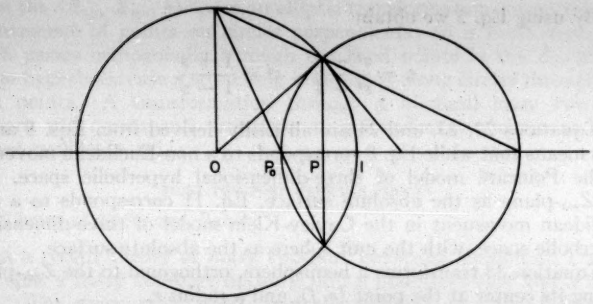

FIG. 5. The Darboux transformation.

with the coordinates (x, y, z) (see Fig. 5) by the following equations:

$$\left.\begin{aligned} x &= \frac{2x_0}{x_0^2 + y_0^2 + z_0^2 + 1} \\ y &= \frac{2y_0}{x_0^2 + y_0^2 + z_0^2 + 1} \\ z &= \frac{2z_0}{x_0^2 + y_0^2 + z_0^2 + 1} \end{aligned}\right\}. \tag{21}$$

A combination of Eqs. 20 and 21 yields the following transformation equations:

$$\left.\begin{aligned} x &= \frac{2R_{cor}}{R^2_{cor} + X^2_{cor} + h^2 + 1} \\ y &= \frac{2X_{cor}}{R^2_{cor} + X^2_{cor} + h^2 + 1} \\ z &= \frac{R^2_{cor} + X^2_{cor} + h^2 - 1}{R^2_{cor} + X^2_{cor} + h^2 + 1} \end{aligned}\right\} \tag{22}$$

which represent the analytic connection between the point Q in the Poincaré model and the point P in the Cayley-Klein model (**13, 14**). The inverse transformation of Eq. 22 is directly:

$$\left.\begin{aligned} R_{cor} &= \frac{x}{1 - z} \\ X_{cor} &= \frac{y}{1 - z} \\ h &= \frac{+}{(-)} \frac{\sqrt{1 - x^2 - y^2 - z^2}}{1 - z} \end{aligned}\right\}. \tag{23}$$

By using Eq. 5 we obtain

$$z = \frac{s^2 - 1}{s^2 + 1}; \quad s^2 = \frac{1 + z}{1 - z}. \tag{24}$$

Equations 22, 23, and 24 are all easily derived from Eqs. 8 and 9. This means that while Eq. 8 corresponds to a non-Euclidean movement in the Poincaré model of three-dimensional hyperbolic space, with the Z_{eor}-plane as the absolute surface, Eq. 11 corresponds to a non-Euclidean movement in the Cayley-Klein model of three-dimensional hyperbolic space, with the unit sphere as the absolute surface.

Equation 23 transforms a hemisphere, orthogonal to the Z_{eor}-plane, having its center at the point (e, f), and a radius r,

$$R^2_{eor} + X^2_{eor} + h^2 - 2eR_{eor} - 2fX_{eor} + e^2 + f^2 - r^2 = 0$$

into a plane cutting the unit sphere of the Cayley-Klein model:

$$ux + vy + wz + 1 = 0 \tag{25}$$

where:

$$\left.\begin{aligned} u &= -\frac{2e}{1 + e^2 + f^2 - r^2} \\ v &= -\frac{2f}{1 + e^2 + f^2 - r^2} \\ w &= \frac{1 - e^2 - f^2 + r^2}{1 + e^2 + f^2 - r^2} \end{aligned}\right\}. \tag{26}$$

Example

The two isometric spheres and the symmetry plane used in the example exemplifying the isometric sphere method above (see Fig. 2) are transformed into the planes:

$$\left.\begin{aligned} y + 2z - 1 &= 0 \\ 4y - z - 1 &= 0 \\ 2y + z - 1 &= 0 \end{aligned}\right\}.$$

The planes all cut along a straight line which is parallel to the x-axis and which cuts the yz-plane in the point $(0, \frac{1}{3}, \frac{1}{3})$. The straight line cuts the surface of the unit sphere in two points constituting the image points obtained by stereographically mapping the fixed points in the Z_{eor}-plane on the unit sphere. (The fixed points are marked by crosses in Fig. 2.) In the example chosen the transformation is a non-Euclidean rotation around this straight line, that is, it is an elliptic transformation (4, 5).

In the (R_{cor}, X_{cor}, h)-space an elliptic transformation corresponds to a movement of points on circles perpendicular to a fixed semicircle which passes orthogonally through the fixed points in the Z_{cor}-plane. In the hyperbolic case a stretching is obtained along circles through the fixed points. A transformation through a bilateral lossy two-port network will correspond, therefore, to a loxodromic movement on a horned cyclide (9, 15).

NOISY TWO-PORT NETWORK THEORY

The Theory of Rothe and Dahlke

In a recent theory of noisy two-port networks Rothe and Dahlke (16) split a noisy two-port network into a noise-free part and a noisy part. The noisy part, which basically consists of a complex noise voltage source V and a complex noise current source I (see Fig. 6), is

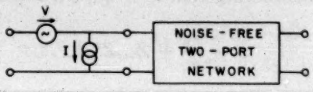

FIG. 6. Splitting of a noisy two-port network into noisy and noise-free parts.

represented by an equivalent circuit which contains an equivalent noise resistance r_n, an equivalent noise conductance g_n, and a complex correlation impedance $Z_{cor} = R_{cor} + jX_{cor}$ (see Fig. 7). Following Rothe and

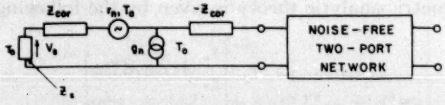

FIG. 7. The equivalent noisy two-port network of Rothe and Dahlke.

Dahlke we split the noise voltage V into two components:

$$V = V_1 + Z_{cor}I, \tag{27}$$

where

$$\overline{V_1 I^*} = 0. \tag{28}$$

Thus V_1 and $Z_{cor}I$ are uncorrelated. By introducing a complex correlation coefficient

$$\gamma = \frac{\overline{VI^*}}{\sqrt{\overline{VV^*}\,\overline{II^*}}} = \frac{Q_2}{\sqrt{Q_1 Q_4}} \tag{29}$$

the authors show that the complex correlation impedance Z_{cor} can be

expressed as:

$$Z_{cor} = \gamma \sqrt{\frac{\overline{VV^*}}{\overline{II^*}}} \tag{30}$$

which is the expression for the correlation impedance used above; that is:

$$Z_{cor} = \frac{Q_2}{Q_4} = \frac{P_1 + jP_2}{P_4 - P_3}. \tag{31}$$

By using the equations

$$\left.\begin{aligned} \overline{V_1 V_1^*} &= 4kT_0 \Delta f r_n \\ \overline{II^*} &= 4kT_0 \Delta f g_n \end{aligned}\right\} \tag{32}$$

where k the Boltzmann constant, T_0 the absolute temperature, Δf the bandwidth, and remembering that $s^2 = Q_1/Q_4$, Eq. 27 yields:

$$s^2 = \frac{r_n}{g_n} + Z_{cor} Z^*_{cor}. \tag{33}$$

A comparison with Eq. 5 shows that

$$h = \sqrt{\frac{r_n}{g_n}}. \tag{34}$$

Therefore the connection between the Rothe and Dahlke theory and the new geometric-analytic theory is given by the following equations:

$$Q = 4kT_0 \Delta f g_n \begin{vmatrix} \frac{r_n}{g_n} + Z_{cor} Z^*_{cor} \\ Z_{cor} \\ Z^*_{cor} \\ 1 \end{vmatrix} \tag{35}$$

and

$$P = 4kT_0 \Delta f g_n \begin{vmatrix} R_{cor} \\ X_{cor} \\ \frac{1}{2}\left(\frac{r_n}{g_n} + Z_{cor} Z^*_{cor} - 1\right) \\ \frac{1}{2}\left(\frac{r_n}{g_n} + Z_{cor} Z^*_{cor} + 1\right) \end{vmatrix}. \tag{36}$$

Equation 36 checks with Eq. 22.

A noise ensemble average transformation through a noise-free two-port network is performed by transforming the complex four-vector Q by Eq. 7 or by transforming the real four-vector P by Eq. 10. The 4×4 matrix L in Eq. 7 is constructed in a regular and simple manner by using the elements of the 2×2 matrix T in Eq. 1. The matrix L is termed the Kronecker or tensor product of T^* by T and it is denoted $T \times T^*$ (17). Thus, Eq. 13 can be written as:

$$Q'' = Q^0 + Q' = Q^0 + T \times T^* Q. \tag{37}$$

The 4×4 matrix M in Eq. 10 belongs to the G_+ subgroup of the proper Lorentz group (5).

Cascading of Noisy Two-Port Networks

If we insert the Q-vectors (35) in Eq. 37 we obtain:

$$\left.\begin{aligned} r_n'' &= r_n^0 + r_n' + \frac{g_n^0 g_n' |Z'_{cor} - Z^0_{cor}|}{g_n^0 + g_n'} \\ g_n'' &= g_n^0 + g_n' \\ Z''_{cor} &= \frac{g_n^0 Z^0_{cor} + g_n' Z'_{cor}}{g_n^0 + g_n'} \end{aligned}\right\}. \tag{38}$$

From Eqs. 37 and 35 we also obtain:

$$\left.\begin{aligned} g_n' &= cc^* r_n + |cZ_{cor} + d|^2 g_n \\ r_n' &= \frac{|ad - bc|^2 r_n g_n}{cc^* r_n + |cZ_{cor} + d|^2 g_n} \\ Z'_{cor} &= \frac{ac^* r_n + (aZ_{cor} + b)(c^* Z^*_{cor} + d^*) g_n}{cc^* r_n + |cZ_{cor} + d|^2 g_n} \end{aligned}\right\}. \tag{39}$$

Equations 38 and 39 are formulas obtained by Dahlke (18). In Dahlke's formulas $a_{11} = d$, $a_{12} = c$, $a_{21} = b$, and $a_{22} = a$.

Noise Tuning and Noise Matching

If a signal source with impedance $Z_s = R_s + jX_s$ is connected to the input of the noisy two-port network (see Fig. 7), then noise tuning and noise matching (16) is obtained when:

$$\left.\begin{aligned} X_s &= -X_{cor} \\ R_s^2 &= R^2_{cor} + h^2 = R^2_{cor} + \frac{r_n}{g_n} \end{aligned}\right\}. \tag{40}$$

Equations 40 constitute a generalization of ordinary tuning and matching to three dimensions (see Fig. 8).

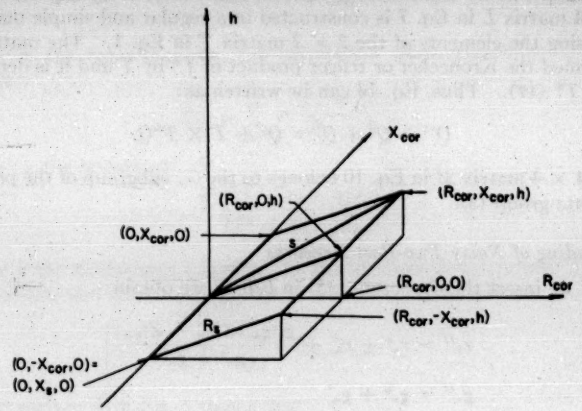

FIG. 8. Noise tuning and noise matching.

Wave Representation

Instead of the voltage-current representation used in previous sections it is sometimes more convenient to use a wave representation.

We split the reflected wave at the output of the two-port network into two components

$$a_2 = a_{2n} + \Gamma_{cor} a_1. \tag{41}$$

a_1 is the transmitted wave and Γ_{cor} is a complex correlation reflection coefficient, defined by

$$\Gamma_{cor} = \frac{\overline{a_2 a_1^*}}{\overline{a_1 a_1^*}} \tag{42}$$

In Eq. 41, a_{2n} and $\Gamma_{cor} a_1$ are uncorrelated, so that

$$\overline{a_{2n} a_1^*} = 0. \tag{43}$$

If we set

$$\begin{aligned} \overline{a_2 a_2^*} &= k\Delta f T_2 \\ \overline{a_{2n} a_{2n}^*} &= k\Delta f T_{2n} \\ \overline{a_1 a_1^*} &= k\Delta f T_1 \end{aligned} \tag{44}$$

where T_2, T_{2n}, and T_1 are noise temperatures, Eq. 41 yields:

$$T_2 = T_{2n} + |\Gamma_{cor}|^2 T_1. \tag{45}$$

The four-vector Q can now be written in the wave representation as:

$$Q_w = \begin{bmatrix} Q_{w1} \\ Q_{w2} \\ Q_{w3} \\ Q_{w4} \end{bmatrix} = \begin{bmatrix} \overline{a_2 a_2^*} \\ \overline{a_2 a_1^*} \\ \overline{a_1 a_2^*} \\ \overline{a_1 a_1^*} \end{bmatrix} = k\Delta f T_1 \begin{bmatrix} \frac{T_{2n}}{T_1} + |\Gamma_{cor}|^2 \\ \Gamma_{cor} \\ \Gamma_{cor}^* \\ 1 \end{bmatrix}. \tag{46}$$

The reflected and transmitted waves at the input of a noise-free two-port network are expressed in the transmitted and reflected waves at the output by the equation:

$$\psi_w' = \begin{pmatrix} a_2' \\ a_1' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_2 \\ a_1 \end{pmatrix} = T_w \psi_w. \tag{47}$$

By using the chain matrix, we obtain:

$$T_w = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = STS^{-1}. \tag{48}$$

For a noise process we have:

$$Q_w' = \begin{bmatrix} \overline{a_2' a_2'^*} \\ \overline{a_2' a_1'^*} \\ \overline{a_1' a_2'^*} \\ \overline{a_1' a_1'^*} \end{bmatrix} = \begin{bmatrix} AA^* & AB^* & BA^* & BB^* \\ AC^* & AD^* & BC^* & BD^* \\ CA^* & CB^* & DA^* & DB^* \\ CC^* & CD^* & DC^* & DD^* \end{bmatrix} \begin{bmatrix} \overline{a_2 a_2^*} \\ \overline{a_2 a_1^*} \\ \overline{a_1 a_2^*} \\ \overline{a_1 a_1^*} \end{bmatrix} = L_w Q_w \tag{49}$$

where

$$L_w = \tfrac{1}{4} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} aa^* & ab^* & ba^* & bb^* \\ ac^* & ad^* & bc^* & bd^* \\ ca^* & cb^* & da^* & db^* \\ cc^* & cd^* & dc^* & dd^* \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}. \tag{50}$$

L_w is the product of the three Kronecker products $S \times S$, $T \times T^*$, and $S^{-1} \times S^{-1}$.

If we set:

$$\left.\begin{aligned} V &= \frac{1}{\sqrt{2}}(a_1 + a_2) \\ I &= \frac{1}{\sqrt{2}}(a_1 - a_2) \end{aligned}\right\} \quad \left.\begin{aligned} a_2 &= \frac{1}{\sqrt{2}}(V - I) \\ a_1 &= \frac{1}{\sqrt{2}}(V + I) \end{aligned}\right\} \tag{51}$$

we obtain

$$\left.\begin{aligned} P_{s1} &= \tfrac{1}{2}(\overline{VI^*} + \overline{V^*I}) = -\tfrac{1}{2}(\overline{a_2a_2^*} - \overline{a_1a_1^*}) = -P_{\Gamma 3} \\ P_{s2} &= -\frac{j}{2}(\overline{VI^*} - \overline{V^*I}) = -\frac{j}{2}(\overline{a_2a_1^*} - \overline{a_1a_2^*}) = P_{\Gamma 2} \\ P_{s3} &= \tfrac{1}{2}(\overline{VV^*} - \overline{II^*}) = \tfrac{1}{2}(\overline{a_2a_1^*} + \overline{a_1a_2^*}) = P_{\Gamma 1} \\ P_{s4} &= \tfrac{1}{2}(\overline{VV^*} + \overline{II^*}) = \tfrac{1}{2}(\overline{a_2a_2^*} + \overline{a_1a_1^*}) = P_{\Gamma 4} \end{aligned}\right\}. \quad (52)$$

Thus Eq. 49 can be interpreted geometrically as a movement in a Poincaré model of three-dimensional hyperbolic space that has the complex reflection coefficient plane for its absolute surface. The corresponding Cayley-Klein model, which has the unit sphere as the absolute surface, is obtained from the Cayley-Klein model treated in previous sections by a 90° rotation around the y-axis.

Wave representations of noisy two-port networks have been studied by Bauer and Rothe (19, 20).

FOUR-DIMENSIONAL TREATMENT

Coherency Matrix

If we post-multiply the vector ψ' in Eq. 1 by its transposed conjugate value, indicated by the symbol †, we obtain:

$$\psi'\psi'^{\dagger} = T\psi(T\psi)^{\dagger} = T\psi\psi^{\dagger}T^{\dagger} \quad (53)$$

or

$$\begin{pmatrix} V'V'^* & V'I'^* \\ V'^*I' & I'I'^* \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} VV^* & VI^* \\ V^*I & II^* \end{pmatrix}\begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}. \quad (54)$$

Equation 54, which has been derived by Cauer (21), for example, can be considered to be the transformation equation for the components and the magnitude of the associate vector of a spinor (22, 23). With a certain correlation between the voltage and the current we can write Eq. 54 in the form:

$$\begin{pmatrix} Q_1' & Q_2' \\ Q_3' & Q_4' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} Q_1 & Q_2 \\ Q_3 & Q_4 \end{pmatrix}\begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix} \quad (55)$$

or

$$C' = \begin{pmatrix} P_4' + P_3' & P_1' + jP_2' \\ P_1' - jP_2' & P_4' - P_3' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} P_4 + P_3 & P_1 + jP_2 \\ P_1 - jP_2 & P_4 - P_3 \end{pmatrix}\begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix} = TCT^{\dagger}. \quad (56)$$

Optical Analogy

At this point it is interesting to compare some of the derived equations with analogous equations in matrix optics. In optics the voltage-current vector ψ corresponds to the Maxwell vector composed of two complex perpendicular fieldstrength components E_x and E_y. The com-

plex four-vector Q in Eq. 7 is analogous to the complex Stokes vector, and the real four-vector P in Eq. 9 is analogous to the real Stokes vector. The Stokes vector was introduced by Stokes (24) in 1852 for representing partially polarized light. The complex 2×2 matrix T in Eq. 1 is analogous in optics to the Jones matrix (25); the complex 4×4 matrix in Eq. 7 and the real 4×4 matrix in Eq. 9 are analogous to 4×4 matrices used by Soleillet (26), Perrin (27), Chandrasekhar (28), Mueller (29, 30), Parke (31–34), McCormick (35), Manasse (36), and others. The Kronecker (tensor) product $T \times T^*$ was introduced in optics in (35).

The 2×2 Hermitian matrix C in Eq. 56 is analogous to the "density matrix" used by von Neumann (37) in quantum mechanics, and to the "coherency matrix" used by Wiener in optics (38–40). The coherency matrix has found extensive application in communication theory (41, 42).

It can be proved by means of the Schwartz inequality that the determinant of the coherency matrices C and C' of Eq. 56 are equal to or greater than zero, that is,

$$\det C = D^2 \geq 0, \quad \det C' = D'^2 \geq 0. \tag{57}$$

We can rewrite Eq. 56 in the form:

$$\sigma_1 P_1' + \sigma_2 P_2' + \sigma_3 P_3' + \sigma_4 P_4' = T(\sigma_1 P_1 + \sigma_2 P_2 + \sigma_3 P_3 + \sigma_4 P_4)T^\dagger \tag{58}$$

where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & j \\ -j & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{59}$$

are the Pauli matrices (43, 44). By using the Pauli matrices, Eqs. 55 and 56 can be interpreted as transformations of Q four-vectors and P four-vectors in four-dimensional spaces.

The determinant D^2 of the coherency matrix C is an important quantity in the theory of noisy two-port networks. We obtain:

$$D^2 = Q_1 Q_4 - Q_2 Q_3 = P_4^2 - P_3^2 - P_2^2 - P_1^2 = (4kT_0\Delta f)^2 r_n g_n. \tag{60}$$

We find, from Eqs. 5, 8, 34, and 60:

$$h = \frac{D}{Q_4} = \frac{D}{P_4 - P_3} = \sqrt{\frac{r_n}{g_n}}. \tag{61}$$

Geometric Interpretation of Hermitian Forms

If, in Eq. 1, instead of postmultiplying ψ' with its transposed conjugate value we premultiply with the same value, we obtain:

$$\psi'^\dagger \psi' = (T\psi)^\dagger T\psi = \psi^\dagger T^\dagger T\psi. \tag{62}$$

Here $T^{\dagger}T$ is a Hermitian matrix which we denote by

$$T^{\dagger}T = \begin{pmatrix} A & B \\ B^* & C \end{pmatrix}. \tag{63}$$

Equations 62 and 63 yield a Hermitian form

$$q(\psi) = A\ V\ V^* + B\ V^*I + B^*VI^* + C\ I\ I^*. \tag{64}$$

In a geometric interpretation of Hermitian forms Deschamps (**44**) sets

$$T^{\dagger}T = -\sigma_1 X - \sigma_2 Y - \sigma_3 Z + \sigma_4 T \tag{65}$$

thus establishing a one-to-one correspondence between the Hermitian matrices and the vectors of a four-dimensional Minkowski space with real coordinates (X, Y, Z, T).

Four-Dimensional Derivation of the Noise Factor

Let us assume that we have split a noisy two-port network into noisy and noise-free parts according to Fig. 7. We then attach a noisy impedance $Z_s = R_s + jX_s$ at the input of the network. The excess noise factor is, by definition,

$$F_s = \frac{Q_1'}{\overline{V_sV_s^*}} = \frac{L_aQ}{\overline{V_sV_s^*}} \tag{66}$$

where L_a is a four-vector composed of the first row of the 4×4 matrix L in Eq. 7. Thus

$$L_a = (aa^*, ab^*, ba^*, bb^*) \tag{67}$$

and

$$\overline{V_sV_s^*} = 4kT\Delta f R_s. \tag{68}$$

Whence the excess noise-factor is proportional to the scalar product of two four-vectors. For the series impedance Z_s, we have $a = 1$, $b = Z_s$, $c = 0$, and $d = 1$, so that

$$F_s = \frac{Q_1 + Z_s^*Q_2 + Z_sQ_3 + Z_sZ_s^*Q_4}{4kT\Delta f R_s} \tag{69}$$

which is a well known expression (**16, 45, 46**). The optimum excess-noise factor is obtained in the usual manner by equating the partial

differentiations with respect to X_s and R_s to zero. Whence we obtain:

$$\left.\begin{aligned} X_{s,\,\mathrm{opt}} &= -\frac{Q_{2i}}{Q_4} = -\frac{P_2}{P_4 - P_3}, \quad Q_2 = Q_{2r} + jQ_{2i} \\ R_{s,\,\mathrm{opt}} &= \frac{\sqrt{Q_1Q_4 - Q_{2i}^2}}{Q_4} = \frac{\sqrt{P_1^2 + D^2}}{P_4 - P_3} \\ F_{s,\,\mathrm{opt}} &= \frac{1}{2kT\Delta f}\left(\sqrt{Q_1Q_4 - Q_{2i}^2} + Q_{2r}\right) = \frac{1}{2kT\Delta f}\left(\sqrt{P_1^2 + D^2} + P_1\right) \end{aligned}\right\}. \quad (70)$$

Theory of Longitudinal Electron Beams

In a theory of longitudinal electron beams Haus (**47, 48**), and Haus and Robinson (**49**) define the self-power density spectrum (SPDS) of the kinetic noise-voltage modulation Φ, the SPDS of the noise-current modulation Ψ and the cross-power density spectrum (CPDS) between the kinetic voltage and current modulations θ. These variables can be combined to form a Q-vector:

$$Q = 4\pi\Delta f \begin{pmatrix} \Phi \\ \theta \\ \theta^* \\ \Psi \end{pmatrix}. \quad (71)$$

CONCLUSION

The geometric-analytic theory of noisy two-port networks shows, that from the standpoint of projective geometry: (1) bilateral lossless two-port networks are essentially one-dimensional in structure and can be described by real numbers; (2) bilateral lossy two-port networks are essentially two-dimensional and can be described by complex numbers; (3) noisy two-port networks are, on an ensemble average ratio basis, essentially three-dimensional and can be described by combinations made up of a complex number and a real number, or of three real numbers, and finally; and (4) noisy two-port networks are, on an ensemble average basis, essentially four-dimensional and can be described by combinations made up of a complex number and two real numbers, or of four real numbers.

The fact that Eq. 8 is a conformal transformation makes extensions to higher dimensional spaces possible. So, for example, the three-dimensional (R_{cor}, X_{cor}, h)-space can be stereographically mapped on the surface of a unit hypersphere (**50**). However, such transformations have not been treated in the paper.

Parts of the geometric-analytic theory of noisy two-port networks have been published in the Quarterly Progress Reports of the Research Laboratory of Electronics, Massachusetts Institute of Technology (July 15, October 15, 1957, and January 15, 1958).

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New Stereophonic Radio System.—A new radio broadcasting system that provides full stereophonic sound through a single receiver and dual speakers on the regular AM broadcast band has been announced by the Radio Corporation of America. The experimental system, described by Dr. James Hillier, Vice President, RCA Laboratories, as "perhaps the longest forward stride in the standard radio broadcast field in nearly thirty years," was demonstrated for the first time to an audience of broadcast station executives at RCA's David Sarnoff Research Center at Princeton, New Jersey.

"With the mounting public interest in stereophonic sound reproduction as a means of achieving the highest fidelity," said Dr. Hillier, "a new system that provides stereo for the first time entirely within the standard AM radio broadcast band is a development of major significance.

"This laboratory system demonstrates the practicability of stereophonic AM broadcasting and reception with a single receiver and a single transmitter operating within the presently assigned frequency of each AM broadcast station.

"Until now, there has been no stereo system that operates entirely in the AM range. This has left a gap in the pattern of stereophonic sound reproduction, since the bulk of commercial broadcasting and listening in this country, including all automobile radios, is in the AM band. With the closing of this gap, we can look forward to the ever more extensive use of stereophonic techniques in our radio programming and listening."

Dr. Hillier described the operation of the RCA Laboratory stereo AM system this way:

Two separate sound channels, picked up by two seperate microphones or by a stereo disk or tape pickup at the studio, are transmitted on the regular broadcast frequency. A standard AM frequency is made up of a carrier wave and two symmetrical sidebands of slightly higher and lower frequencies to either side of the carrier. In the stereo system demonstrated to the broadcasters, each of the stereo channels is carried by one of these sidebands.

In the stereophonic AM receiver, the two sidebands are separated and fed to two speakers, left and right, to reproduce the stereo effect picked up at the studio. In a present conventional AM receiver, there would be no separation of the two sidebands, so that the program would be heard in conventional fashion without the stereo effect. The stereophonic receiver also can pick up non-stereo broadcasts and play them through either speaker or both, without any stereophonic effect.

Dr. Hillier pointed to the following advantages of a practical stereophonic AM broadcast system, adding that all of these advantages are present in the new technique demonstrated by RCA:

1. Stereophonic music and other program material, either live or recorded, can be sent from a single transmitter operating within the present AM frequency of a broadcasting station.

2. Stereophonic reception is accomplished with a single receiver feeding into matched loudspeakers that can be brought into desired balance with a single control.

3. Since the system operates in the regular AM broadcast band, it can be used for automobile radios, for which FM systems are not practical.

4. The stereophonic system is adapted to present AM broadcast techniques, so that it could be introduced without causing obsolescence of present conventional receivers.

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A GREEN'S FUNCTION SOLUTION FOR THE CASE OF LAMINAR INCOMPRESSIBLE FLOW BETWEEN NON-CONCENTRIC CIRCULAR CYLINDERS

BY

JAMES F. HEYDA [1]

ABSTRACT

The main result is the determination of Green's function in bi-polar coordinates for the potential equation for a non-concentric annular region. This is then used to solve Poisson's equation for the point velocity in laminar incompressible flow between non-concentric circular cylinders.

I. INTRODUCTION

An explicit formulation for the case of a concentric circular ring of Green's function for the potential equation is given by Hilbert-Courant (1).[2] When the bounding circles of the annular region are no longer concentric, the reflection principle used in (1) is no longer practical. Upon introducing bi-polar coordinates in the manner suggested in Morse-Feshbach (2), a fairly simple form for Green's function can be obtained. This result is then used to solve Poisson's equation for the point velocity in laminar incompressible flow between non-concentric circular cylinders. When the eccentricity of the annular region tends to zero, this velocity formulation reduces to the well-known result for a concentric annulus.

II. BI-POLAR COORDINATES

In Fig. 1, if P is distant r_1 from the fixed pole $O_1(c, 0)$ and r_2 from the pole $O_2(-c, 0)$ and the angles of the vectors O_1P and O_2P are θ_1 and θ_2, respectively, then P is defined to have the bi-polar coordinates (ξ, η) where

$$\xi = \pi - (\theta_1 - \theta_2), \quad \eta = \log(r_1/r_2). \tag{1}$$

The curves $\xi = \text{const}$ are a family of circles passing through O_1 and O_2, the ξ-values for points on the circle segments below the x-axis being π more than the ξ-values for points on the corresponding circle segments above the x-axis.

The curves $\eta = \text{const}$ are also a family of circles, having centers on the x-axis and being normal to all the ξ-circles. For $\eta < 0$, the circles lie in the right half plane and enclose O_1, while for $\eta > 0$ the circles lie in the left half plane and enclose O_2.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

If we prefer, we may regard the xy-plane in Fig. 1 as the complex z-plane and then define a new complex variable w by $w = \xi + i\eta$. To

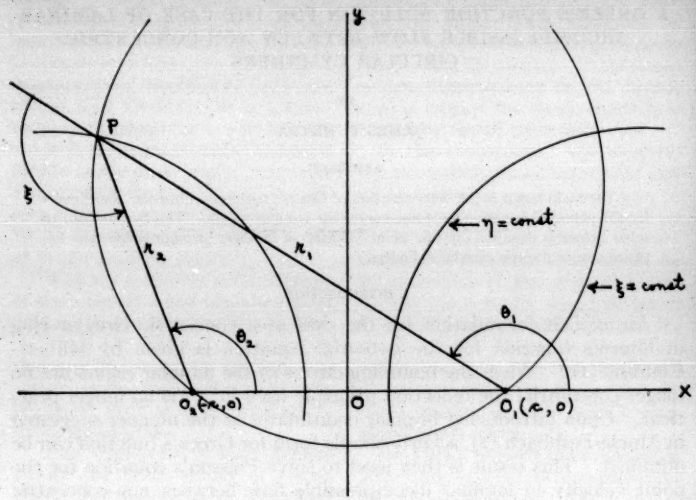

FIG. 1.

determine the transformation between the z and w planes implied by the bi-polar relations (1), observe that

$$\frac{r_1}{r_2} = e^{\eta} = \frac{|\overrightarrow{PO} + \overrightarrow{OO_1}|}{|\overrightarrow{O_2O} + \overrightarrow{OP}|} = \frac{|c - z|}{|c + z|};$$

or,

$$e^{-\eta} = \frac{|c + z|}{|c - z|}.$$

Similarly,

$$\xi = \theta_2 - (\theta_1 - \pi) = \arg(c + z) - \arg(c - z) = \arg\left(\frac{c + z}{c - z}\right).$$

Hence, ξ and $e^{-\eta}$ are respectively the angle and the absolute value of the complex variable

$$\zeta = \frac{c + z}{c - z}. \tag{2}$$

Writing ζ in polar form

$$\zeta = \rho e^{i\xi}, \quad \rho = e^{-\eta}, \tag{3}$$

we have

$$\zeta = e^{-\eta+i\xi} = e^{i(\xi+i\eta)} = e^{iw},$$

whence

$$z = c\left(\frac{e^{iw}+1}{e^{iw}-1}\right). \tag{4}$$

From (2) we see that in the ζ-plane, the circle $|\zeta| = e^{-\eta}$ is the map of the circle $\left|\frac{c+z}{c-z}\right| = e^{-\eta}$, which, in the z-plane, has center at $(-c\coth\eta, 0)$ and is of radius $c\,\mathrm{csch}\,\eta$.

Applying these remarks to the non-concentric annular region of interest shown in Fig. 2, we may regard the given circles A, B, with

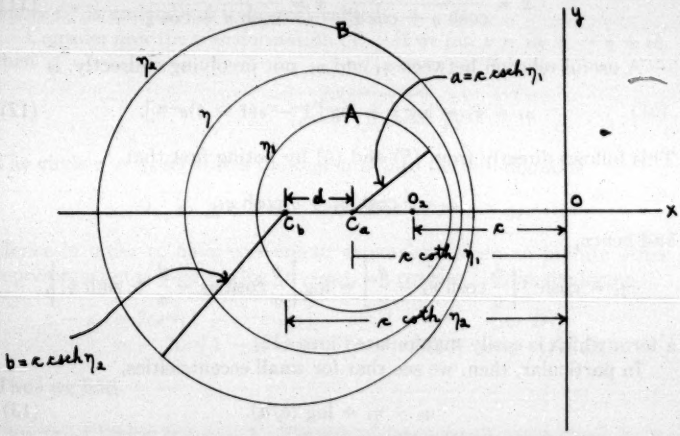

FIG. 2.

radii a, b, respectively, as two particular members of the η-family of circles enclosing point $O_2\ (-c, 0)$. If we assign A and B the values η_1 and η_2, then knowledge of a, b and d, where d is the distance between the centers C_a and C_b, is sufficient to determine η_1, η_2 and c. To see this, observe that

$$a = c\,\mathrm{csch}\,\eta_1, \quad b = c\,\mathrm{csch}\,\eta_2 \tag{5}$$

$$d = c\,(\coth\eta_2 - \coth\eta_1), \tag{6}$$

three equations which may be solved for the unknowns η_1, η_2, c. If we define the eccentricity ϵ and the radius ratio s by

$$\epsilon = d/(b - a), \quad s = a/b, \tag{7}$$

we find that these unknowns may be computed in the order η_2, c, η_1 from

$$\eta_2 = \cosh^{-1} \tfrac{1}{2} \left\{ \frac{1 + s}{\epsilon} + (1 - s)\epsilon \right\}, \tag{8}$$

$$c = b \sinh \eta_2 \tag{9}$$

$$\eta_1 = \sinh^{-1} (c/a). \tag{10}$$

Equation 4 yields the transformation equations

$$x = \frac{-c \sinh \eta}{\cosh \eta + \cos \xi}, \quad y = \frac{c \sin \xi}{\cosh \eta + \cos \xi}. \tag{11}$$

A useful relation between η_1 and η_2, not involving c directly, is

$$\eta_1 = \eta_2 - \log s + \log [1 - \epsilon(1 - s)e^{-\eta_1}]. \tag{12}$$

This follows directly from (5) and (6) by noting first that

$$d = b \cosh \eta_2 - a \cosh \eta_1$$

and hence,

$$\eta_1 = \cosh^{-1} \left[\frac{b}{a} \cosh \eta_2 - \frac{d}{a} \right] = \log \left[\frac{b}{a} \cosh \eta_2 - \frac{d}{a} + \sinh \eta_1 \right],$$

a form which is easily manipulated into (12).

In particular, then, we see that for small eccentricities,

$$\eta_1 - \eta_2 \doteq \log (b/a). \tag{13}$$

To effect a reduction to the concentric case, we let $d \to 0$, whence

$$\epsilon \to 0, \quad c \to \infty, \quad \eta_1 \to \infty, \quad \eta_2 \to \infty,$$

but

$$\lim (\eta_1 - \eta_2) = \log (b/a). \tag{14}$$

III. DERIVATION OF GREEN'S FUNCTION

Let (x, y) and (x_0, y_0) be distinct points of the non-concentric annular region D shown in Fig. 2. Then, it will be recalled, the Green's

function in question is the function $G(x, y; x_0, y_0)$, having the following properties:

(*a*) $\nabla^2 G = 0$ in D for all $(x, y) \neq (x_0, y_0)$, when G is regarded as a function of (x, y).

(*b*) For points (x, y) on the boundaries A or B, we have $G = 0$.

(*c*) $G(x, y; x_0, y_0) = -\frac{1}{2\pi} \log \sqrt{(x-x_0)^2+(y-y_0)^2} + G^*(x, y; x_0, y_0)$, so that G has a logarithmic singularity at (x_0, y_0), while the function G^* is analytic in D without exception and $\nabla^2 G^* = 0$.

In terms of the complex variables $z = x + iy$, $z_0 = x_0 + iy_0$, we may write G as

$$G(z, z_0) = -\frac{1}{2\pi} \operatorname{Re} \log (z - z_0) + G^*(z, z_0), \tag{15}$$

where G^* is understood to be real-valued.

Consider now the transformation (4). If we put $u = iw = -\eta + i\xi$, then

$$z - z_0 = \frac{2c(e^{u_0} - e^u)}{(e^u - 1)(e^{u_0} - 1)}. \tag{16}$$

The circle $\eta = \eta_0$ separates the region D into the sub-domains

$$D_1\colon\ \eta_1 \geq \eta > \eta_0 \quad \text{and} \quad D_2\colon\ \eta_0 > \eta \geq \eta_2.$$

Hence in order to have convergent expansions when an infinite series representation is used for $\log (z - z_0)$, we rewrite (16) in the forms

$$\begin{aligned} z - z_0 &= 2ce^{u_0}(1 - e^{u-u_0})/(1 - e^u)(1 - e^{u_0}), \quad (\text{for } D_1) \\ &= -2ce^u[1 - e^{-(u-u_0)}]/(1 - e^u)(1 - e^{u_0}), \quad (\text{for } D_2). \end{aligned}$$

Thus we find

$$\log (z-z_0) = \log 2c + u_0 - \sum_{n=1}^{\infty} \frac{1}{n}[e^{n(u-u_0)} - (e^{nu} + e^{nu_0})], \quad \eta_1 \geq \eta > \eta_0$$

$$= i\pi + \log 2c + u - \sum_{n=1}^{\infty} \frac{1}{n}[e^{-n(u-u_0)} - (e^{nu} + e^{nu_0})], \quad \eta_0 > \eta \geq \eta_2.$$

Therefore upon taking real parts we find

$$-\frac{1}{2\pi} \operatorname{Re} \log (z - z_0)$$

$$= \frac{1}{2\pi}\left[\eta_0 - \log 2c + \sum_{n=1}^{\infty} \frac{1}{n}\{e^{-n(\eta-\eta_0)} \cos n(\xi - \xi_0) - (e^{-n\eta} \cos n\xi + e^{-n\eta_0} \cos n\xi_0)\}\right] \tag{17a}$$

when $\eta_1 \geq \eta > \eta_0$, and

$$-\frac{1}{2\pi}\operatorname{Re}\log(z-z_0)$$
$$=\frac{1}{2\pi}\left[\eta-\log 2c+\sum_{n=1}^{\infty}\frac{1}{n}\{e^{n(\eta-\eta_0)}\cos n(\xi-\xi_0)-(e^{-n\eta}\cos n\xi+e^{-n\eta_0}\cos n\xi_0)\}\right] \quad (17b)$$

when $\eta_0 > \eta \geq \eta_2$.

Returning now to Eq. 15, we denote Green's function in bi-polar coordinates by $G(\xi, \eta; \xi_0, \eta_0)$ and its non-singular component by $G^*(\xi, \eta; \xi_0, \eta_0)$. For convenience we also denote (17) by $S(\xi, \eta; \xi_0, \eta_0)$. Thus Eq. 17 becomes

$$G(\xi, \eta; \xi_0, \eta_0) = S(\xi, \eta; \xi_0, \eta_0) + G^*(\xi, \eta; \xi_0, \eta_0). \quad (18)$$

Since $\nabla^2 G^* = 0$, the most general G^* in terms of n^{th} order circular harmonics is

$$G^* = A_0\eta + B + \sum_{n=1}^{\infty}[(A_n\cos n\xi + B_n\sin n\xi)e^{-n\eta} + (C_n\cos n\xi + D_n\sin n\xi)e^{n\eta}]. \quad (19)$$

The constants in (19) can now be evaluated by noting that $G(\xi, \eta; \xi_0, \eta_0)$ must vanish when $\eta = \eta_1$ and also when $\eta = \eta_2$. We then have the conditions

$$G^*(\xi, \eta_1; \xi_0, \eta_0) = -S(\xi, \eta_1; \xi_0, \eta_0),$$
$$G^*(\xi, \eta_2; \xi_0, \eta_0) = -S(\xi, \eta_2; \xi_0, \eta_0).$$

Using these conditions, we obtain, after considerable simplification, the following form of Green's function for the domains D_1 and D_2.

For D_1: $\eta_1 \geq \eta > \eta_0 \geq \eta_2$, (20)

$$G(\xi, \eta; \xi_0, \eta_0) = \frac{1}{2\pi}\left[\frac{(\eta_1-\eta)(\eta_0-\eta_2)}{\eta_1-\eta_2}+\sum_{n=1}^{\infty}\{e^{-n(\eta-\eta_0)}-H(\eta,\eta_0;\eta_1,\eta_2)\}\frac{\cos n(\xi-\xi_0)}{n}\right];$$

For D_2: $\eta_1 \geq \eta_0 > \eta \geq \eta_2$,

$$G(\xi, \eta; \xi_0, \eta_0) = \frac{1}{2\pi}\left[\frac{(\eta-\eta_2)(\eta_1-\eta_0)}{\eta_1-\eta_2}+\sum_{n=1}^{\infty}\{e^{n(\eta-\eta_0)}-H(\eta,\eta_0;\eta_1,\eta_2)\}\frac{\cos n(\xi-\xi_0)}{n}\right],$$

where

$$H(\eta, \eta_0; \eta_1, \eta_2) = \frac{e^{-n(\eta-\eta_2)}\sinh n(\eta_1-\eta_0) - e^{n(\eta-\eta_1)}\sinh n(\eta_2-\eta_0)}{\sinh n(\eta_1-\eta_2)}. \quad (21)$$

It should be noted that the series

$$\sum_{n=1}^{\infty}\frac{1}{n}e^{-n(\eta-\eta_0)}\cos n(\xi-\xi_0), \quad \sum_{n=1}^{\infty}\frac{1}{n}e^{n(\eta-\eta_0)}\cos n(\xi-\xi_0)$$

diverge when $(\xi_0, \eta_0) = (\xi, \eta)$, as they should, since $G(\xi, \eta; \xi_0, \eta_0)$, considered as a function of (ξ, η), has a logarithmic-type singularity at (ξ_0, η_0). It is also readily verified that $G(\xi, \eta_i; \xi_0, \eta_0) \equiv 0, \quad i = 1, 2.$

In examining Eqs. 20 one notes that the symmetry property,

$$G(\xi, \eta; \xi_0, \eta_0) \equiv G(\xi_0, \eta_0; \xi, \eta),$$

which is characteristic of Green's functions, is not readily apparent. This is easily remedied by noting that

$$e^{-n(\eta-\eta_0)} - H \equiv \frac{2 \sinh n(\eta_1 - \eta) \sinh n(\eta_0 - \eta_2)}{\sinh n(\eta_1 - \eta_2)},$$

$$e^{n(\eta-\eta_0)} - H \equiv \frac{2 \sinh n(\eta - \eta_2) \sinh n(\eta_1 - \eta_0)}{\sinh n(\eta_1 - \eta_2)}.$$

Thus, the Green's function takes on the compact, symmetrical and rather elegant form

For D_1: $\eta_1 \geq \eta > \eta_0 \geq \eta_2$, $\quad G \equiv G_1$, where (22)

$$G_1(\xi, \eta; \xi_0, \eta_0) = \frac{1}{2\pi}\left[\frac{(\eta_1 - \eta)(\eta_0 - \eta_2)}{\eta_1 - \eta_2} + 2\sum_{n=1}^{\infty} \frac{\sinh n(\eta_1 - \eta) \sinh n(\eta_0 - \eta_2)}{n \sinh n(\eta_1 - \eta_2)} \cos n(\xi - \xi_0)\right];$$

For D_2: $\eta_1 \geq \eta_0 > \eta \geq \eta_2$, $\quad G \equiv G_2$, where

$$G_2(\xi, \eta; \xi_0, \eta_0) = \frac{1}{2\pi}\left[\frac{(\eta - \eta_2)(\eta_1 - \eta_0)}{\eta_1 - \eta_2} + 2\sum_{n=1}^{\infty} \frac{\sinh n(\eta_1 - \eta_0) \sinh n(\eta - \eta_2)}{n \sinh n(\eta_1 - \eta_2)} \cos n(\xi - \xi_0)\right].$$

IV. POINT VELOCITY IN LAMINAR FLOW

The laminar flow of an incompressible fluid of viscosity μ through the region bounded by non-concentric circular cylinders is described by Poisson's equation

$$\nabla^2 v = -\frac{1}{\mu}\frac{dp}{dL}, \tag{23}$$

where v is the point velocity at (ξ, η) in a normal cross section D and dp/dL is the pressure gradient at D in the direction of the flow.

Let $\frac{1}{\mu} dp/dL$ be denoted by the constant K. Then stated as a boundary value problem, we wish to solve

$$\nabla^2 v = -K, \tag{24}$$

subject to the conditions $v(\xi, \eta_1) = 0$, $v(\xi, \eta_2) = 0$, where η_1, η_2 characterize the boundary circles A, B of the region D.

Since we now have the Green's function available, the solution can

be written down directly. Thus,

$$v(\xi, \eta) = K \int\int_D G(\xi, \eta; \xi_0, \eta_0) dA \tag{25}$$

where the element of area $dA = dxdy = J(x, y/\xi_0, \eta_0) d\xi_0 d\eta_0$, the Jacobian being computed from the transformation Eqs. 11. We find

$$dA = \frac{c^2 d\xi_0 d\eta_0}{(\cosh \eta_0 + \cos \xi_0)^2}, \tag{26}$$

so that the solution (25), expressed as an iterated integral, becomes

$$v(\xi, \eta) = Kc^2 \int_{\eta_1}^{\eta_2} \int_0^{2\pi} \frac{G(\xi, \eta; \xi_0, \eta_0) d\xi_0 d\eta_0}{(\cosh \eta_0 + \cos \xi_0)^2}. \tag{27}$$

In terms of G_1 and G_2 as defined in (22), this may in turn be written

$$v(\xi, \eta) = Kc^2 \int_{\eta_1}^{\eta} \int_0^{2\pi} \frac{G_2(\xi, \eta; \xi_0, \eta_0) d\xi_0 d\eta_0}{(\cosh \eta_0 + \cos \xi_0)^2} + Kc^2 \int_{\eta}^{\eta_2} \int_0^{2\pi} \frac{G_1(\xi, \eta; \xi_0, \eta_0) d\xi_0 d\eta_0}{(\cosh \eta_0 + \cos \xi_0)^2}. \tag{28}$$

We write (28) as

$$v(\xi, \eta) = I_1 + I_2 + I_3 + I_4,$$

where

$$I_1 = \frac{Kc^2(\eta - \eta_2)}{2\pi(\eta_1 - \eta_2)} \int_{\eta_1}^{\eta} \int_0^{2\pi} \frac{\eta_1 - \eta_0}{(\cosh \eta_0 + \cos \xi_0)^2} d\xi_0 d\eta_0, \tag{30}$$

$$I_2 = \frac{Kc^2(\eta_1 - \eta)}{2\pi(\eta_1 - \eta_2)} \int_{\eta}^{\eta_2} \int_0^{2\pi} \frac{\eta_0 - \eta_2}{(\cosh \eta_0 + \cos \xi_0)^2} d\xi_0 d\eta_0, \tag{31}$$

$$I_3 = \frac{Kc^2}{\pi} \sum_{n=1}^{\infty} \int_{\eta_1}^{\eta} \int_0^{2\pi} \frac{\sinh n(\eta_1 - \eta_0) \sinh n(\eta - \eta_2) \cos n(\xi - \xi_0) d\xi_0 d\eta_0}{n \sinh n(\eta_1 - \eta_2) (\cosh \eta_0 + \cos \xi_0)^2}, \tag{32}$$

$$I_4 = \frac{Kc^2}{\pi} \sum_{n=1}^{\infty} \int_{\eta}^{\eta_2} \int_0^{2\pi} \frac{\sinh n(\eta_1 - \eta) \sinh n(\eta_0 - \eta_2) \cos n(\xi - \xi_0) d\xi_0 d\eta_0}{n \sinh n(\eta_1 - \eta_2) (\cosh \eta_0 + \cos \xi_0)^2}. \tag{33}$$

The integrals I_1 and I_2 are readily evaluated and their sum is

$$I_1 + I_2 = -\frac{Kcd}{2} \left(\frac{\eta_1 - \eta}{\eta_1 - \eta_2} \right) + \frac{Kc^2}{2} (\coth \eta - \coth \eta_1). \tag{34}$$

The integration with respect to ξ_0 in (32) and (33) is a matter of

evaluating the integral

$$I^* = \frac{1}{2\pi}\int_0^{2\pi} \frac{\cos n(\xi - \xi_0)}{(\cosh \eta_0 + \cos \xi_0)^2}\, d\xi_0. \tag{35}$$

Employing symmetry considerations and putting $z = e^{i\xi_0}$, we may write (35) as a contour integral,

$$I^* = \frac{2 \cos n\xi}{\pi i} \oint \frac{z^{n+1} dz}{(z + e^{\eta_0})^2 (z + e^{-\eta_0})^2}, \tag{36}$$

the integration being taken around the unit circle $|z| = 1$. Since $\eta_0 > 0$ the integrand in (36) has a pole of order 2 at $-e^{-\eta_0}$. Thus we may write

$$I^* = 4 \cos n\xi \left\{ \frac{d}{dz}\left[\frac{z^{n+1}}{(z + e^{\eta_0})^2} \right] \right\}_{z = -e^{-\eta_0}},$$

where

$$I^* = (-1)^n e^{-n\eta_0} \left[\frac{\cosh \eta_0 + n \sinh \eta_0}{\sinh^3 \eta_0} \right] \cos n\xi. \tag{37}$$

Using (37) the remaining integrations in (32), (33) are easily made. Combining the results for I_3, I_4 so obtained with (34), we obtain the result

$$\begin{aligned} v(\xi, \eta) = &- \frac{Kcd}{2}\left(\frac{\eta_1 - \eta}{\eta_1 - \eta_2} \right) + \frac{Kc^2}{2} (\coth \eta - \coth \eta_1) \\ &+ Kcd \sum_{n=1}^{\infty} (-1)^n e^{-n\eta_1} \frac{\sinh n(\eta - \eta_2)}{\sinh n(\eta_1 - \eta_2)} \cos n\xi \\ &- Kc^2 \left[\sum_{n=1}^{\infty} (-1)^n e^{-n\eta} \cos n\xi \right] (\coth \eta_2 - \coth \eta). \end{aligned} \tag{38}$$

It is easily verified that $v(\xi, \eta_1) \equiv v(\xi, \eta_2) \equiv 0$ if one uses relation (6).

The form (38) may be simplified somewhat by using the following relation, derived in (2, p. 1215).

$$1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-n\eta} \cos n\xi = \frac{\sinh \eta}{\cosh \eta + \cos \xi}.$$

With additional simplifications we can then show that

$$\begin{aligned} v(\xi, \eta) = &- \frac{Kcd}{2}\left(\frac{\eta_1 - \eta}{\eta_1 - \eta_2} \right) + \frac{Kc^2}{2} \frac{\sinh \eta (\coth \eta - \coth \eta_1)}{\cosh \eta + \cos \xi} \\ &- Kcd \sum_{n=1}^{\infty} (-1)^n e^{-n\eta_2} \frac{\sinh n(\eta_1 - \eta)}{\sinh n(\eta_1 - \eta_2)} \cos n\xi. \end{aligned} \tag{39}$$

If we let R be the radius of the circle with parameter η, then using (14) we see that as the annular eccentricity $\epsilon \to 0$,

$$\frac{\eta_1 - \eta}{\eta_1 - \eta_2} \to \frac{\log (R/a)}{\log (b/a)}.$$

We note also that, although $c \to \infty$ and $d \to 0$ the product

$$cd \to \tfrac{1}{2}(b^2 - a^2).$$

This follows since

$$cd = c^2[\coth \eta_2 - \coth \eta_1]$$

$$= c^2\left[\sqrt{1 + \frac{b^2}{c^2}} - \sqrt{1 + \frac{a^2}{c^2}}\right] = \frac{b^2 - a^2}{\sqrt{1 + \frac{b^2}{c^2}} + \sqrt{1 + \frac{a^2}{c^2}}}.$$

Thus

$$-\frac{Kcd}{2}\left(\frac{\eta_1 - \eta}{\eta_1 - \eta_2}\right) \to -\frac{K}{4}(b^2 - a^2)\frac{\log (R/a)}{\log (b/a)}. \tag{40}$$

Similarly we can show that as $\epsilon \to 0$

$$\frac{Kc^2}{2}\frac{\sinh \eta(\coth \eta - \coth \eta_1)}{\cosh \eta + \cos \xi} \to \frac{K}{4}(R^2 - a^2).$$

Finally, we note that

$$\left[\sum_{n=1}^{\infty} (-1)^n e^{-n\eta_2} \cos n\xi \frac{\sinh n(\eta_1 - \eta)}{\sinh n(\eta_1 - \eta_2)}\right] < \frac{e^{-\eta_2}}{1 - e^{-\eta_2}} \to 0.$$

Thus, in the limit as $\epsilon \to 0$, the form (39) tends towards the well-known solution for the concentric annulus, namely,

$$v(x, y) = -\frac{K}{4}\left[R^2 - a^2 - (b^2 - a^2)\frac{\log (R/a)}{\log (b/a)}\right].$$

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CONSTANT-RESISTANCE ALL-PASS NETWORKS WITH MAXIMALLY FLAT TIME DELAY *

BY

LOUIS WEINBERG [1]

INTRODUCTION

A practical need often arises for a lumped-constant time-delay network. Modern synthesis satisfies this need quite well. To allow the results of synthesis to be used by engineers possessing little or no synthesis background, tables of ladder networks with maximally flat delay were given in two previous papers (1, 2).[2] However, engineers using these tables have stated that they had other requirements to be satisfied by the delay network. They wanted one or both of the following properties:

1. Tap-off points at prescribed fractions of the total delay or, what is equivalent, the possibility of adding delay networks in tandem without using an intermediate isolating amplifier.
2. A magnitude characteristic that was uniform throughout the linear range of the phase function.

The answer to these requirements is given by a *constant-resistance all-pass* network with maximally flat delay. The writer therefore referred the engineers to the excellent paper by Thomson (3) in which a complete solution to this problem is given. To obtain the desired networks all that is necessary is an understanding of Thomson's paper, a subsequent Foster or Cauer design of an inductance-capacitance (LC) driving-point function, and the use of duality to realize another impedance.

However, this did not end the matter: the answer was not satisfactory to most of the practical engineers. It appears that there is a large body of engineers who want a final working formula and a set of tables. It is true that most of these engineers would find no difficulty with the partial-fraction expansion required by the Foster design, but when the required expansion is set in a context of Bessel and Lommel polynomials, and the necessity of solving for the roots of such polynomials of high degree, then it may become formidable. Thus some elegant networks are neglected by the practical engineer. This paper is

* This paper is based on the author's report with the same title, Technical Memorandum No. 473, Hughes Research Laboratories, Culver City, Calif. A condensed version of part of the paper was published in *Electronic Design*, September 15, 1957.

[1] Hughes Research Laboratories, Culver City, Calif.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

an attempt to make these networks available with little or no computation. The tables in the paper are directed primarily to those engineers without synthesis background. It is hoped, moreover, that the tables will be valuable to the synthesis expert since they allow him to devote his available computing time and facilities to the difficult network problems for which tables do not suffice.

The tables give the element values of the series arms of a lattice network that yields a maximally flat time delay; the element values of the cross arms are obtained by duality. The lattice may be driven by a current source with a shunt conductance or a voltage source with a series resistance; that is, by use of the tables we may realize a transfer impedance $Z_{21} = E_2/I_1$, a transfer admittance $Y_{21} = I_2/E_1$, or a transfer voltage ratio $K = E_2/E_1$.

The lattice that is obtained is a *constant-resistance all-pass* network. By constant-resistance is meant that the input impedance is constant for all frequencies. This has two desirable features. The first is that a generator with a finite and nonzero internal resistance may be used as

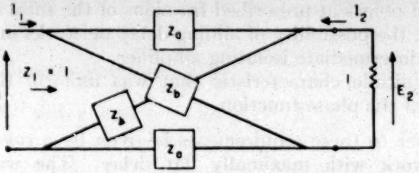

FIG. 1. Circuit diagram of constant-resistance lattice terminated in one ohm.

the source without changing the desired transfer function except for its constant multiplier. Secondly, lattices with the same value of constant-resistance input can be cascaded without interaction effects. Thus, as in a true transmission line, taps may be made at desired values of time delay. The all-pass designation means that the lattices do not affect the magnitude versus frequency characteristic of the input signal. In short, for signals whose frequency spectrum does not extend beyond the linear range of the phase function of the network, the conditions for facsimile reproduction are met, that is, the output time function is of the same shape as the input time function, but delayed by the value of the time delay of the network.

The maximally flat time-delay network has been discussed previously (1–5), and its properties are therefore not repeated here. In the first part of the paper we discuss how to use the tables to derive the impedances of the lattice arms, and show that alternate realizations are possible. In the second part a few illustrative examples are worked out. The Appendix is devoted to the derivation of the formulas used in the synthesis of these constant-resistance all-pass lattices.

USE OF THE TABLES

A constant-resistance lattice with a one-ohm termination is shown in Fig. 1.[3] The two driving-point impedances labeled Z_a are called series arms and the two labeled Z_b are called cross arms. Because of the constant-resistance property

$$Z_a Z_b = 1. \tag{1}$$

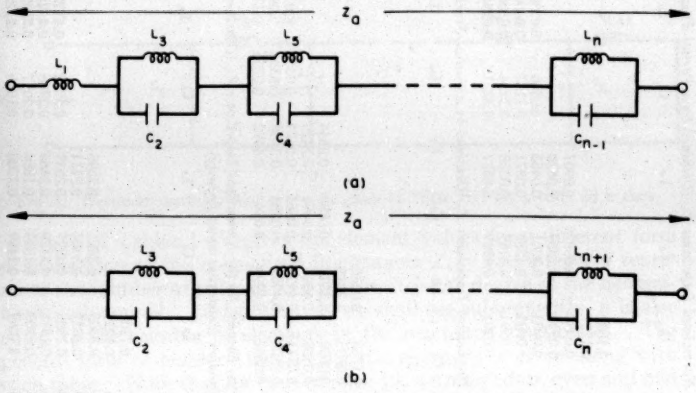

FIG. 2. Networks corresponding to the Z_a given in Table I: (*a*) n odd; (*b*) n even.

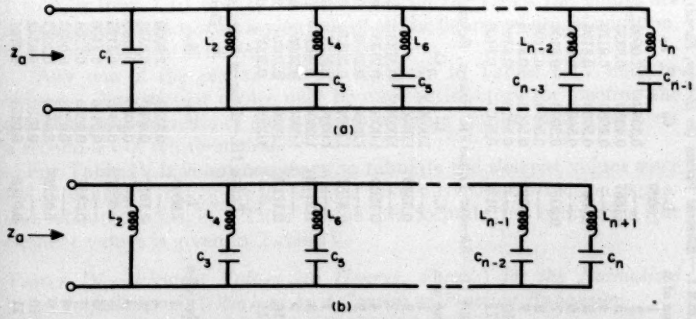

FIG. 3. Networks corresponding to the Z_a given in Table II: (*a*) n even; (*b*) n odd.

[3] For clarity, the two equal series-arm impedances and the two equal cross-arm impedances have been included in the figure. It is conventional, however, to omit drawing one impedance of each pair, as is done in Fig. 7.

TABLE I.—*Element Values (in Henrys, Farads) for the Normalized Impedance Z_a (Obtained by a Partial-Fraction Expansion of Z_a).*

Value of n	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9	C_{10}	L_{11}
1	1.0000										
2	——	0.3333	1.0000								
3	0.1667	0.4800	0.8333								
4	——	0.4988	0.8120	0.1251	0.1880						
5	0.0667	0.5000	0.8106	0.3192	0.1227						
6	——	0.5000	0.8106	0.4439	0.0988	0.0597	0.0907				
7	0.0357	0.5000	0.8106	0.4889	0.0918	0.1839	0.0620				
8	——	0.5000	0.8106	0.4985	0.0903	0.3201	0.0453	0.0346	0.0538		
9	0.0222	0.5000	0.8106	0.4999	0.0901	0.4239	0.0371	0.1137	0.0400		
10	——	0.5000	0.8106	0.5000	0.0901	0.4769	0.0338	0.2154	0.0300	0.0226	0.0356
11	0.0152	0.5000	0.8106	0.5000	0.0901	0.4949	0.0327	0.3221	0.0234	0.0767	0.0281

TABLE II.—*Element Values (in Henrys, Farads) for the Normalized Impedance Z_a (Obtained by a Partial-Fraction Expansion of $1/Z_a$).*

Value of n	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8	C_9	L_{10}	C_{11}	L_{12}
1	——	1.0000										
2	0.3333	1.0000										
3	——	1.0000	0.3333	0.2000								
4	0.1000	1.0000	0.2333	0.4082								
5	——	1.0000	0.2078	0.4841	0.1255	0.0838						
6	0.4762	1.0000	0.2032	0.4983	0.0825	0.2418						
7	——	1.0000	0.2027	0.4999	0.0620	0.3843	0.0687	0.0446				
8	0.2778	1.0000	0.2026	0.5000	0.0538	0.4644	0.0491	0.1429				
9	——	1.0000	0.2026	0.5000	0.0513	0.4925	0.0361	0.2624	0.0433	0.0277		
10	0.1818	1.0000	0.2026	0.5000	0.0508	0.4989	0.0284	0.3736	0.0333	0.0925		
11	——	1.0000	0.2026	0.5000	0.0507	0.4999	0.0247	0.4492	0.0256	0.1786	0.0298	0.0188

TABLE III.—*Element Values (in Henrys, Farads) for the Normalized Impedance Z_a (Obtained by a Continued-Fraction Expansion about Infinity).*

Value of n	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9	C_{10}	L_{11}
1	1.0000										
2	——	0.3333	1.0000								
3	0.1667	0.4800	0.8333								
4	——	0.1000	0.2899	0.4627	0.7101						
5	0.0667	0.1948	0.3103	0.4215	0.6231						
6	——	0.0476	0.1400	0.2246	0.3005	0.3821	0.5595				
7	0.0357	0.1055	0.1704	0.2288	0.2827	0.3487	0.5111				
8	——	0.0278	0.0823	0.1338	0.1806	0.2227	0.2639	0.3212	0.4732		
9	0.0222	0.0660	0.1077	0.1463	0.1811	0.2129	0.2465	0.2986	0.4424		
10	——	0.0182	0.0541	0.0886	0.1209	0.1549	0.1880	0.2057	0.2209	0.2712	0.4161
11	0.0152	0.0451	0.0741	0.1016	0.1269	0.1499	0.1708	0.1916	0.2175	0.2639-	0.3955

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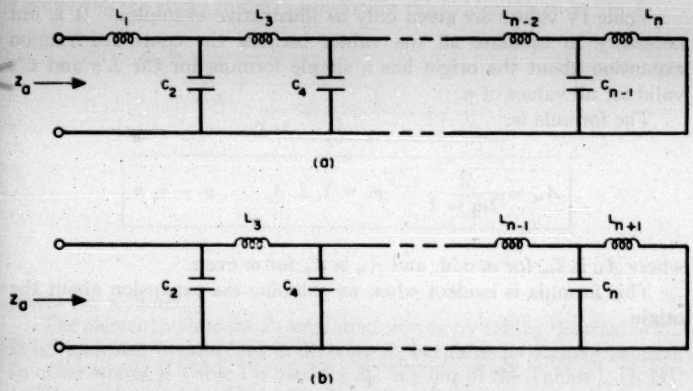


FIG. 4. Networks corresponding to the Z_a given in Table III: (*a*) n odd; (*b*) n even.

Each of Tables I–IV gives the element values for a different form of realization of the normalized impedances Z_a. The integer n represents the number of poles in Z_{21}, that is, it is the degree of the denominator polynomial. In addition, as we shall see subsequently, n is also equal to the number of elements in the reactance function Z_a. The general form of network identifying the elements is given along with each table. Note that for convenience (to accommodate even and odd values of n with only one scheme of notation for each table) the subscripts identifying the elements in Figs. 2–4 for Tables I–III run from 1 to n or from 2 to $n + 1$. As indicated on the tables the values are in henrys and farads; this is also true of all the figures where, in addition, the termination is in ohms.

Any one of the realizations for Z_a given in Tables I–IV may be chosen. A particular choice may be most satisfactory for meeting the practical requirements of one problem whereas another may best meet a second set of requirements.

For Table IV it is not necessary to tabulate the element values since for all values of n they are given by a known continued-fraction expansion about the origin. Thus the simple formula for computing the element values is given in Table IV.

TABLE IV.—*Element Values (in Henrys, Farads) for the Normalized Impedance Z_a (Obtained by a Continued-Fraction Expansion About the Origin).*

Value of n	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9	C_{10}	L_{11}
1	1.0000										
2	1.0000	0.3333									
3	1.0000	0.3333	0.2000								

Table IV values are given only as illustrative examples. It is not necessary to tabulate all the values because the continued-fraction expansion about the origin has a simple formula for the L's and C's valid for all values of n.

The formula is

$$A_m = \frac{1}{2m-1} \qquad m = 1, 2, 3, \cdots, n-1, n$$

where A_m is L_m for m odd, and A_m is C_m for m even.

This formula is evident when we examine the expansion about the origin

$$Z_a = \cfrac{1}{\cfrac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{1}{\cfrac{5}{s} + \cdots + \cfrac{1}{\cfrac{2n-1}{s}}}}}$$

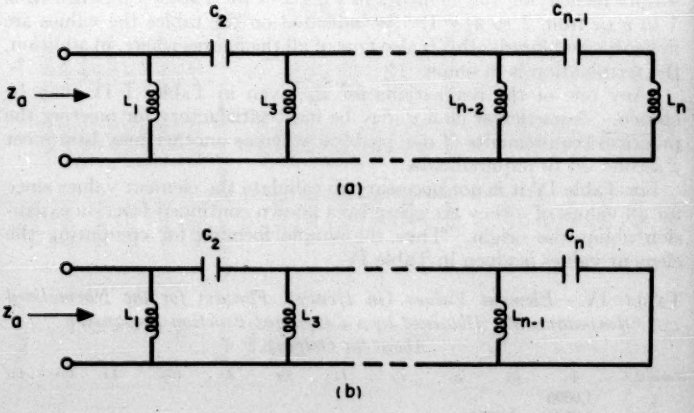


FIG. 5. Networks corresponding to the Z_a given in Table IV: (*a*) n odd; (*b*) n even.

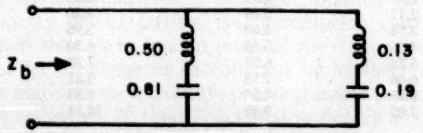
Comparing the above to the expansion for the ladders given in Fig. 5,

$$Z_a = \cfrac{1}{\cfrac{1}{L_1 s} + \cfrac{1}{\cfrac{1}{C_2 s} + \cfrac{1}{\cfrac{1}{L_3 s} + \cfrac{1}{\cfrac{1}{C_4 s} + \cfrac{1}{\cfrac{1}{L_5 s} + \cdots}}}}}$$

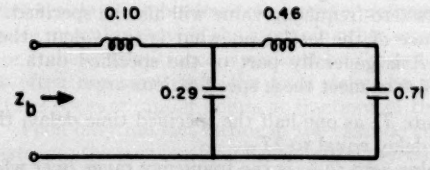
yields the formula.

The element values for Z_b are found simply by taking the dual of Z_a. It is important to note that in deriving Z_b *any* table for Z_a may be used. In other words, if Table I is used for Z_a, any one of the Tables I, II, III, or IV may be used to determine Z_b.

To obtain the dual or reciprocal of Z_a, an inductance of L henrys becomes a capacitance of L farads, and conversely a capacitance of C farads becomes an inductance of C henrys. Series connections become parallel connections, and conversely. Thus the reciprocals of Z_a given by Tables I and III for $n = 4$ are given, respectively, by Figs. 6*a* and 6*b*.



(a)



(b)

FIG. 6. Networks for Z_b obtained by reciprocation.

The element values obtained for Z_a and Z_b in this manner are normalized with respect to the load resistance R and the time delay T_0, that is, they apply for $R = 1$ and for $\omega T_0 = 1$. (Thus the frequency is normalized with respect to $\omega_0 \equiv 1/T_0$. However, as shown in the Appendix, the lattice provides a delay of $2T_0$ at zero frequency.) To convert the lattice with a one-ohm termination to a lattice with a termination of R ohms (and thus an input impedance of R ohms) we multiply every inductance value by R and divide every capacitance by R, that is, we raise the impedance level of the network by R. To remove the normalization with respect to time delay we consider the desired zero-frequency time delay equal to $2T_0$; then T_0 is equal to one half of the desired delay. Every capacitance and inductance is now divided by the radian frequency $\omega_0 \equiv 1/T_0$. The removal of the two normalizations can be combined into a single step: multiply every inductance by $R/\omega_0 = RT_0$ and every capacitance by $1/(R\omega_0) = T_0/R$.

In Table V are listed the values of frequency u for which the time

TABLE V.—*Significant Values of u for Time Delay of a Maximally Flat Time-Delay Network (Giving Frequencies (u) at Which Time Delay Deviates a Specified Value from its Zero-Frequency Value).*

n	u for 1% deviation	u for 10% deviation	u for 20% deviation	u for 50% deviation
1	0.10	0.34	0.50	1.00
2	0.56	1.09	1.39	2.20
3	1.21	1.94	2.29	3.40
4	1.93	2.84	3.31	4.60
5	2.71	3.76	4.20	5.78
6	3.52	4.69	5.95	6.97
7	4.36	5.64	6.30	8.15
8	5.22	6.59	7.30	9.33
9	6.08	7.55	8.31	10.50
10	6.96	8.52	9.33	11.67
11	7.85	9.49	10.34	12.84

delay deviates a specified percentage from its zero-frequency value. The variable u is defined as $\omega T_0 = \omega/\omega_0$ and is thus a normalized radian frequency.

In a typical problem the frequency band over which a specified time delay is desired will be given, and the permissible deviation of the time delay from its zero-frequency value will also be specified. Finally, the input resistance of the lattice or, what is equivalent, the value of the termination R is generally part of the specified data. The steps in using the tables to meet these specifications are:

1. Compute T_0 as one half the specified time delay, that is, set the desired time delay equal to $2T_0$.

2. Assuming zero to ω_1 is the frequency range over which the specified time delay is required, compute $u_1 = \omega_1 T_0$.

3. With b representing the allowable percentage deviation of the time delay choose the smallest value of the tabulated deviations—1%,

10%, 20%, 50%—that is not less than b. Now in the column of Table V that corresponds to this choice find the smallest value of $u \geq u_1$. Note the corresponding value of n.

4. Use the n determined above and any one of the Tables I–IV to find the element values of Z_a. Using the configuration given with the selected table draw the network for Z_a.

5. Determine Z_b geometrically as the reciprocal of the Z_a given by *any* one of the Tables I–IV.

6. Now remove the resistance and time-delay normalizations by multiplying every inductance in Z_a and Z_b by RT_0 and every capacitance by T_0/R. The lattice obtained is shown in Fig. 7; it is a constant-resistance all-pass network with a maximally flat time delay.

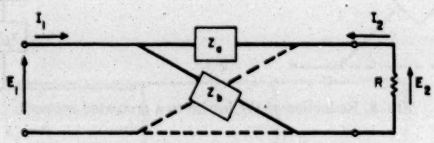


FIG. 7. Resistance-terminated lattice network.

The transfer function that has been realized corresponds to an input I_1 connected directly to the terminals of the lattice, that is, an ideal current source has been assumed. However, because of the constant-resistance property of the lattice, this assumption causes no loss in generality: when the source has an internal resistance, the same Z_{21} is realized except for a change in its constant multiplier. For example, if the lattice is matched for maximum power transfer to a current source I with a shunt resistance R, then I_1 is equal to $I/2$ by a current-divider relationship, so that the over-all transfer impedance is

$$Z_{21}' = \frac{E_2}{I} = \frac{E_2}{2I_1} = \tfrac{1}{2}Z_{21}. \tag{2}$$

Also by a Thévenin transformation of the input current source and its associated shunt resistance it becomes clear that a voltage source with a series resistance may also provide the input; in other words, a transfer voltage ratio or transfer admittance is realized by the same lattice.

It is most often necessary that the total delay be realized by a cascade of lattices because tap-off points at fractions of the total delay are required. Then only the last lattice in the cascade chain is terminated in a resistance; the termination of each of the preceding lattices is automatically provided by the input impedance of the lattice at its output terminals in parallel with the input impedance of the tap-off network. The modification in the procedure for such problems is straightforward and is made clear by the illustrative samples.

Two final points are the introduction of dissipation into Z_a and Z_b and the reduction of the achieved lattice into a grounded bridged-T network. These are merely mentioned here. It is possible to take account of the losses in the coils without changing the frequency characteristic by introducing a flat loss (6). Also, if a grounded network is desired, the lattice transformation of Fig. 8 may be used (7). Then if

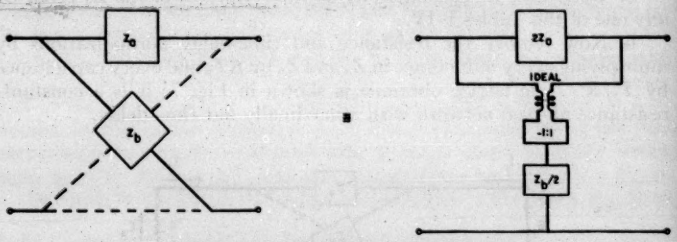


FIG. 8. Reduction of the lattice to a grounded network.

Z_a has a shunt inductance branch and Z_b a series inductance the ideal transformer may be converted to a real transformer by incorporation of these inductances.

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ILLUSTRATIVE EXAMPLES

In this part two examples are given to illustrate the use of the tables.

Example 1

An all-pass constant-resistance lattice is desired with a time delay of one millisecond over a frequency range of 0 to 1000 cycles/sec. The time delay is not to deviate more than 10 per cent over this frequency range. The lattice is to provide maximum power transfer to its load. A cathode follower, which approximates quite closely a voltage source with an internal resistance of 200 ohms, is the input to the lattice.

From the given data

$$2T_0 = 10^{-3}$$

$$\therefore \quad T_0 = 0.5 \times 10^{-3}$$

and

$$\omega_1 = 2\pi \times f_1 = 6.28 \times 10^3.$$

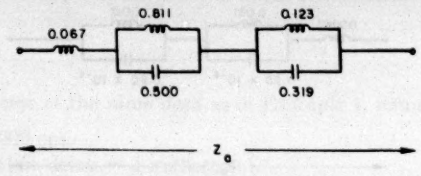
Thus

$$u_1 = \omega_1 T_0 = 3.14.$$

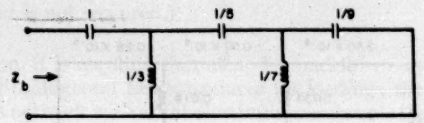
Consulting Table V for 10 per cent deviation we find $u_1 = 3.14$ lies between $n = 4$ and $n = 5$; we therefore need $n = 5$.

Now since any one of Tables I–IV may be used for Z_a and Z_b inde-

pendently, there are 16 possible networks provided by the tables. We show two of these networks. The first obtains Z_a from Table I and Z_b as the reciprocal of the Z_a of Table IV; these impedances are

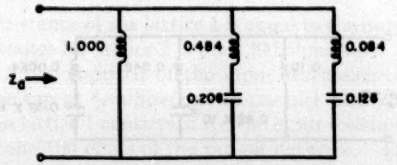


(a)

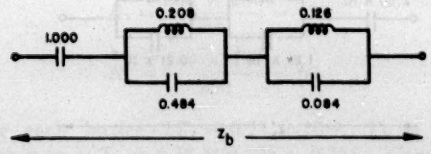


(b)

FIG. 9. Normalized impedances of first lattice network.



(a)



(b)

FIG. 10. Normalized impedances of second lattice network.

shown in Fig. 9. The second has both its Z_a and Z_b given by Table II; these are given in Fig. 10.

The normalizations are now removed by multiplying every induc-

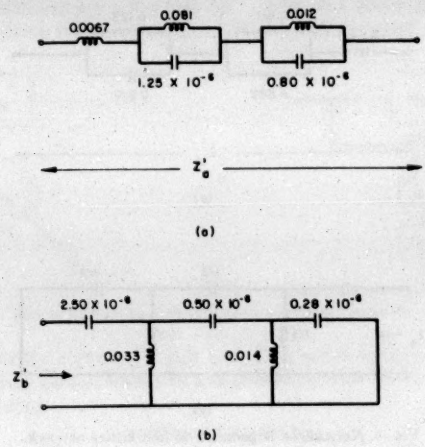


FIG. 11. Impedances of first lattice network after normalizations have been removed.

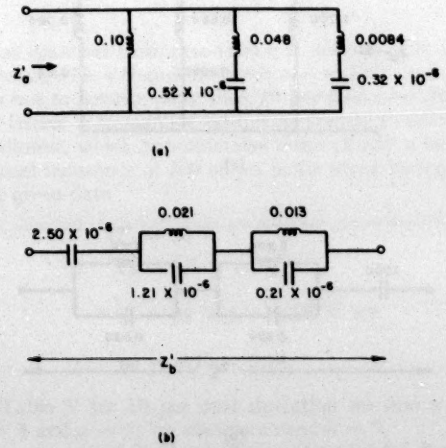


FIG. 12. Impedances of second lattice network after normalizations have been removed.

tance by $RT_0 = 200 \times 0.5 \times 10^{-3} = 0.1$ and every capacitance by $T_0/R = 0.5 \times 10^{-3}/200 = 2.5 \times 10^{-6}$. The final impedances Z_a' and Z_b' for the first and second lattices are shown respectively in Figs. 11 and 12. These impedances are the arms of a lattice terminated in 200 ohms.

Example 2

Assume some of the same data as in Example 1, namely:

(*a*) $f_1 = 1000$ cps.
(*b*) Total time delay = 1 millisecond.
(*c*) Maximum deviation in total delay = 10%.
(*d*) Input is a voltage source with an internal resistance of 200 ohms.
(*e*) The load resistance is 200 ohms. (However, maximum power transfer is not required.)

In addition, it is required that a tap be made at every $\frac{1}{4}$-millisecond delay. The $\frac{3}{4}$-millisecond tap introduces no loading; the $\frac{1}{2}$-millisecond tap is fed to a network with a 600-ohm input resistance; the $\frac{1}{4}$-millisecond tap feeds a network with a 100-ohm input resistance.

Because of the three tap-off points four lattices are required. These lattices will be identified by the numbers 1, 2, 3, and 4, beginning with the output end.

The input resistance of the lattice 1 is equal to the output resistance. The design resistance for lattice 2 is then 200 ohms also. However, the load on lattice 3 is comprised of the input resistance of the network connected to the tap at $\frac{1}{2}$-millisecond in parallel with 200 ohms; similarly, the load on lattice 1 consists of R_3, the input resistance of lattice 3, in parallel with the 100 ohms of the tap-off network. Therefore

$$R_1 = 200$$

$$R_2 = 200$$

$$R_3 = \frac{600 \times 200}{800} = 150$$

$$R_4 = \frac{150 \times 100}{250} = 60.$$

Since for each lattice $2T_0 = 0.25 \times 10^{-3}$, T_0 is 0.125×10^{-3} second. Also $u_1 = \omega_1 T_0 = 6.28 \times 0.125 = 0.785$. Consulting Table V for 10 per cent deviation we find that 2 is the smallest acceptable value of n. (For this small value of n the networks given by continued-fraction or partial-fraction expansions are the same; that is, all the

tables yield the same Z_a.) Using Table III for Z_a and Z_b, we get the normalized impedances of Fig. 13. To remove the normalizations we

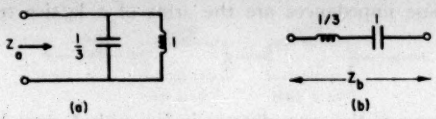


FIG. 13. Normalized impedance arms of lattice networks.

multiply the inductances of the respective lattices by

$$R_1T_0 = R_2T_0 = 200 \times 0.125 \times 10^{-3} = 2.5 \times 10^{-2}$$
$$R_3T_0 = 150 \times 0.125 \times 10^{-3} = 1.875 \times 10^{-2}$$
$$R_4T_0 = 60 \times 0.125 \times 10^{-3} = 7.5 \times 10^{-3}$$

and the capacitances by

$$T_0/R_1 = T_0/R_2 = 0.125 \times 10^{-3}/200 = 6.25 \times 10^{-7}$$
$$T_0/R_3 = 0.125 \times 10^{-3}/150 = 8.33 \times 10^{-7}$$
$$T_0/R_4 = 0.125 \times 10^{-3}/60 = 2.08 \times 10^{-6}.$$

The impedances of the lattices are given respectively by Figs. 14–16. The complete network is shown in Fig. 17.

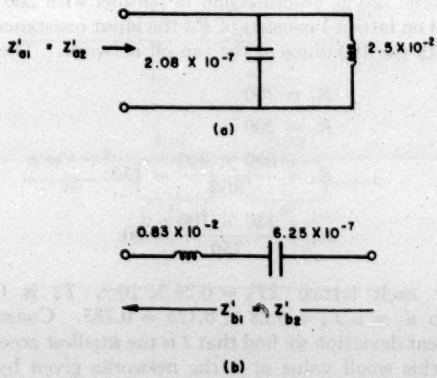


FIG. 14. Impedances of the arms of lattices 1 and 2.

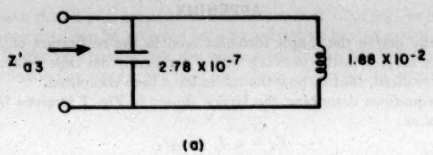


FIG. 15. Impedances of the arms of lattice 3.

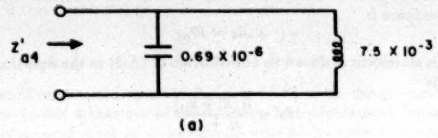


FIG. 16. Impedances of the arms of lattice 4.

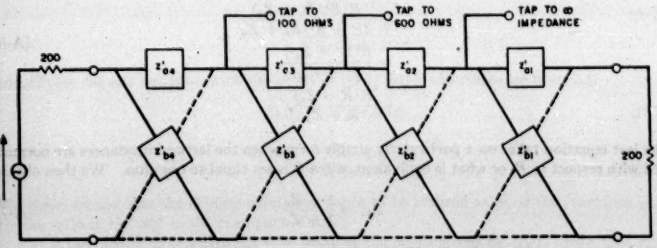


FIG. 17. Network realization for the data of Example 2.

APPENDIX

Here we briefly derive the simple formulas used in the realization of $Z_{21} = E_2/I_1$ as a constant-resistance all-pass lattice network with a maximally flat time delay. Thus we show how the lattice is realized, that is, how the tables have been computed.

The pair of equations describing the lattice shown in Fig. 7 is given in terms of open-circuit impedances as

$$\begin{aligned} E_1 &= z_{11}I_1 + z_{12}I_2 \\ E_2 &= z_{21}I_1 + z_{22}I_2. \end{aligned} \tag{A-1}$$

By straightforward calculation we obtain

$$\begin{aligned} z_{11} &= z_{22} = \tfrac{1}{2}(Z_b + Z_a) \\ z_{21} &= z_{12} = \tfrac{1}{2}(Z_b - Z_a) \\ |z| &= z_{11}z_{22} - z_{12}^2 = Z_aZ_b \\ Z_1 &= \frac{|z| + z_{11}R}{R + z_{22}} = \frac{Z_aZ_b + z_{11}R}{R + z_{22}}, \end{aligned} \tag{A-2}$$

where $|z|$ is the determinant of the matrix of open-circuit impedances, and Z_1 is the input impedance of the lattice.

The necessary and sufficient condition for the lattice to possess the constant-resistance property is that the driving-point impedances of the lattice arms be dual to each other within a scale factor, or stated more precisely, that they be reciprocal functions with respect to R^2; in symbols, the condition is

$$Z_aZ_b = R^2. \tag{A-3}$$

The validity of this statement is shown by substitution of (A-3) in the equation for Z_1 given in (A-2). This yields

$$\begin{aligned} Z_1 &= \frac{R(R + z_{11})}{R + z_{22}} \\ &= R, \end{aligned} \tag{A-4}$$

where use has been made of the fact that $z_{11} = z_{22}$.

The transfer impedance of a quadrupole terminated in a resistance R may be easily calculated as

$$Z_{21} = \frac{E_2}{I_1} = \frac{Rz_{21}}{R + z_{22}} \tag{A-5}$$

which becomes for the constant-resistance lattice

$$\begin{aligned} Z_{21} &= \frac{R(Z_b - Z_a)}{2R + Z_b + Z_a} \\ &= \frac{R(R^2/Z_a - Z_a)}{2R + R^2/Z_a + Z_a} \\ &= \frac{R(R^2 - Z_a^2)}{(R + Z_a)^2} \\ &= \frac{R(R - Z_a)}{R + Z_a}. \end{aligned} \tag{A-6}$$

The last equation takes on a particularly simple form when the lattice impedances are normalized with respect to R, or what is equivalent, when R is set equal to one ohm. We then obtain

$$Z_{21} = \frac{1 - Z_a}{1 + Z_a} \tag{A-7}$$

and the input impedance given in (A-4) becomes one ohm.

For Z_{21} to have the all-pass property it is necessary that $|Z_{21}(j\omega)|$ be constant at all frequencies. This property can be achieved only by a function having a zero at the negative of each pole, that is, the points at which the poles occur in the left half-plane are the mirror images of the corresponding zeros in the right half-plane. Thus for the maximally flat property

$$Z_{21} = \frac{h_n(-s)}{h_n(s)}, \tag{A-8}$$

where $h_n(s)$ are the polynomials derived from the Bessel polynomials. The polynomials h_n have been tabulated in a previous paper (8).

For the constant-resistance lattice we have from (A-7),

$$|Z_{21}(j\omega)|^2 = \frac{(1 - R_a)^2 + X_a^2}{(1 + R_a)^2 + X_a^2}, \tag{A-9}$$

where we have used $Z_a(j\omega) = R_a + jX_a$. The only way to satisfy the requirement that $|Z_{21}|^2$ be constant at all frequencies is to have $R_a = 0$, that is, $Z_a(j\omega) = jX_a$, a pure reactance function. Therefore for an all-pass constant-resistance lattice

$$Z_{21}(j\omega) = \frac{1 - jX_a}{1 + jX_a} \tag{A-10}$$

and the phase is given by

$$\beta = -2 \tan^{-1} X_a. \tag{A-11}$$

The time delay is therefore equal to

$$-\frac{d\beta}{d\omega} = \frac{2}{1 + X_a^2} \frac{dX_a}{d\omega}. \tag{A-12}$$

The method of relating the specified $Z_{21}(s)$ in (A-8) to the design equation for the lattice given in (A-7) or (A-10) makes use of the following fact: the ratio of the even and odd parts of a Hurwitz polynomial (that is, a polynomial with all its zeros in the left half-plane) is a reactance function. The polynomial $h_n(s)$ is of course a Hurwitz one.

It is now convenient to write

$$h_n(s) = M(s) + N(s), \tag{A-13}$$

where M and N are the respective even and odd parts of h_n. Therefore, by use of the definition of even and odd parts,

$$h_n(-s) = M(s) - N(s). \tag{A-14}$$

We next substitute (A-13) and (A-14) in (A-8) and divide numerator and denominator by $M(s)$:

$$Z_{21} = \frac{\dfrac{M(s) - N(s)}{M(s) + N(s)}}{\dfrac{1 - N(s)/M(s)}{1 + N(s)/M(s)}}. \tag{A-15}$$

Identifying the last equation with the terms in (A-7) yields the reactance function,

$$Z_a(s) = N(s)/M(s) \tag{A-16}$$

or

$$\begin{aligned} Z_a(j\omega) &= jX_a(\omega) \\ &= N(j\omega)/M(j\omega). \end{aligned} \tag{A-17}$$

We therefore see that the driving-point impedance to be realized as Z_a is the reactance given by the ratio of the odd to the even part of $h_n(s)$.

It is interesting to compare the time delay of the lattice given by (A-12) with that given by the ladder network whose transfer function is the constant a_0 divided by $h_n(s)$ with the same

value of n,

$$Z_{21} = \frac{a_0}{h_n(s)} = \frac{a_0}{M(s) + N(s)}. \tag{A-18}$$

The phase of this function is the negative of the phase of the denominator. Therefore

$$\beta = -\tan^{-1}\left(\frac{N(j\omega)}{jM(j\omega)}\right) = -\tan^{-1} X_a \tag{A-19}$$

and

$$-\frac{d\beta}{d\omega} = \frac{1}{1 + X_a^2}\frac{dX_a}{d\omega}. \tag{A-20}$$

Thus the time delay of the lattice is exactly twice that of the ladder network with the same number of poles. Tables of the element values for these ladder networks were presented previously (1, 2).

Now since X_a is an odd polynomial in ω divided by an even polynomial, $X_a(0) = 0$. Letting $dX_a/d\omega$ at zero frequency be T_0, we then obtain the zero-frequency time delay as $2T_0$. The delay decreases monotonically from this value for increasing ω.

Finally, it should be clear from Eqs. A-7 and A-3 that the design of the normalized lattice requires only the realization of an LC driving-point function Z_a, and then the reciprocation of Z_a to obtain $Z_b = 1/Z_a$.

There are four basic (or canonic) methods for realizing Z_a (9). One is a partial-fraction expansion of Z_a; this is called a Foster form and yields a series connection of an inductance and parallel LC networks, as shown in Fig. 18*a* for n odd. The second method is a partial-fraction

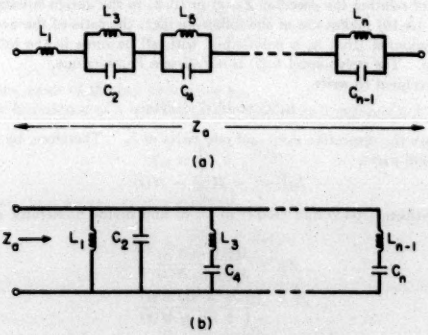


FIG. 18. Foster networks for realizing Z_a.

expansion of $Y_a = 1/Z_a$; this is the second Foster form and yields a parallel connection of branches containing an inductance, a capacitance, and series LC combinations, as shown in Fig. 18*b* for n even.

The third and fourth methods of realization are called Cauer forms. The third is obtained by a continued-fraction expansion of Z_a about infinity; the network obtained is a ladder with series inductances and shunt capacitances. This is shown in Fig. 19*a*, where n has been assumed odd. The second form of Cauer network is a ladder with series capacitances and shunt inductances; this is obtained by a continued-fraction expansion of Z_a about the origin. However, it is not necessary to compute this expansion for each value of n, since the general

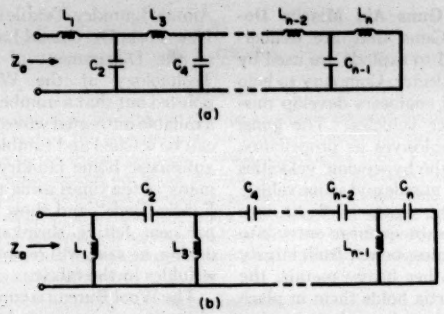


FIG. 19. Cauer networks for realizing Z_a.

form of the expansion is known. The network is shown in Fig. 19*b*, where n has been assumed even.

The element values for each of these realizations are given in the first four tables.

It should be mentioned that it is also possible to use a mixed Foster-Cauer realization; that is, we can carry out a few steps of a continued-fraction expansion and then realize the remainder function by a partial-fraction expansion. Alternatively, we can find a few of the terms in a partial-fraction expansion and then expand the remainder function in a continued fraction. These mixed forms have not been tabulated.

Acknowledgment

The author expresses his thanks to the members of the Mathematics Section, Systems Analysis Department, Hughes Aircraft Company, who carried through almost all of the calculations for this paper.

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(8) L. WEINBERG, "Network Design by Use of Modern Synthesis Techniques and Tables," Technical Memorandum No. 427, Hughes Research Laboratories, Culver City, Calif.; also published in *Proceedings of the National Electronics Conference*, Vol. 12, 1956. It is pointed out that an error occurs in Table 4.3 for $h_n(s)$: for $n = 10$, a_7 should be 25740 (instead of 25750).

(9) L. WEINBERG, "Network Synthesis," a section in the book "Control Engineers' Handbook," edited by J. G. Truxal. New York, McGraw-Hill Book Co., Inc., 1958.

Exploding Guns Aid Missile Development.—Guns that are deliberately designed to explode are used by the General Electric Company to help scientists and engineers develop missiles and space vehicles. The guns, using high explosives as propellants, help provide the hypersonic velocities needed to test missile and space vehicle models for the rigors of flight into space and for subsequent re-entry into the Earth's atmosphere. Built largely of lead and other heavy metals, the guns' own inertia holds them in place and prevents their charge from causing excessive repercussions when the guns explode.

Constructed very simply, a typical gun is built as a deep, heavy-walled, lead cup that is partially filled with explosives. Lining the cup and sealing off the explosives is a heavy, lead sleeve. Into this sleeve is inserted the barrel. The barrel, usually steel, must be greater in density than the projectile, usually aluminum. The projectile, to be constantly supported by the barrel, must move forward faster than the barrel expands.

Developed as a hypersonic testing tool by Dr. F. J. Willig, manager of special projects, and Howard W. Semon, explosive specialist, who are members of the General Electric Aerosciences Laboratory in Philadelphia, the Company's Missile and Space Vehicle Department has been awarded a contract by the Army's Aberdeen Proving Ground to study the gun's possibilities as a research and testing tool.

Washable Wool Suits.—A man's washable all-wool suit is a thing of the not-too-distant future. And when it does become an actuality, it will no doubt be a better garment than any of today's so-called "wash 'n wear" suits. Making this statement at the First Annual Laundry-Textile Conference in New York, Dr. Gerald Laxer, Director of the Department of Science and Technology at the Wool Bureau, pointed out that a number of currently available untreated woven wool fabrics can be washed and tumble dried in the automatic home laundry machine as many as ten times using the "Delicate Fabric Cycle" and show less than a 3 per cent felting shrinkage. Tumble drying, he said, will remove accidental wrinkles in the fabric.

The Wool Bureau is currently supervising experiments at the Harris Research Laboratories in Washington, D. C. where scientists are discovering the principles of construction which make for highly shrink-resistant wool fabrics. These principles are being applied to create new wool fabrics specially for the washable wool garment of the future.

Studies of the tightness of yarn twist and number of yarns per inch in the warp and filling of shrink resistant fabrics indicate that increased twist and increased number of yarns per inch in the warp and filling of wool fabrics will result in less felting shrinkage, without use of chemical treatment.

Studies at Harris Research Laboratories are being amplified so that more detailed knowledge can be obtained on the effects that wool fiber diameter, length and crimp have on felting shrinkage.

Dr. Laxer described the coming "easy-care" wool suit as being completely mothproof, stainproof and washable when necessary without loss of the soft hand and fine appearance for which wool is noted. It will also have creases or pleats which will last the life of the garment.

Producing washable wool garments will require a whole new technology of tailoring, according to Dr. Laxer who said that these are some of the details now being worked on.

V.26
1959

AN ELASTOPLASTIC ANALYSIS OF A ROTATING ANNULUS

BY

M. J. OWEN [1]

SUMMARY

The problem of an elastoplastic rotating annulus is analyzed completely. The analysis is based on the assumption of plastically incompressible material obeying Hooke's Law in the elastic region and yielding according to Tresca's yield criterion. The plastic displacements are found through the flow rule associated with Tresca's yield criterion. The boundary conditions are satisfied for the deformed boundaries. The equations involved are non-linear and a solution is found through a perturbation method. The displacement solution resulting from the use of the flow rule is new, and it is shown that in some cases it is important to satisfy the boundary conditions for the deformed boundaries.

INTRODUCTION

The rotating annular disk is an important machine element and the elastic stress and displacement solution are well known. Disks are often overspeeded during initial testing with the result that they may become partly plastic. Overspeeding may be carried out for many reasons, but often one of them is to introduce favorable residual stresses in the disk. It is thus important to know the elastoplastic solution.

Several solutions have been published in the literature. In 1928 Nadai and Donnell (1)[2] reported solutions based on both Tresca's and v. Mises' yield criteria. The displacement solution is based on deformation theory and an assumption of total incompressibility. In the stress solution elastic compressibility is allowed for and in the numerical examples a value of Poisson's ratio $\nu = 0.3$ is used. This is not compatible with the displacement solution which requires $\nu = 0.5$. Much of the solution is obtained numerically and has to be repeated for each diametral ratio. In 1952 Zaid (2) presented a solution based on v. Mises' yield criterion. Again the displacement solution is based on deformation theory. Zaid (3) and Weiss and Prager (4) have given solutions based on flow theory for fully plastic disks. It is generally believed at the present time that the flow theory is physically more sound than the deformation theory.

The present paper is concerned with an elastoplastic analysis of a rotating annulus of initially uniform thickness. The solution will be based on Tresca's yield criterion and its associated flow rule (5). The variation in thickness due to deformation will be taken into account and the boundary conditions satisfied for the displaced boundaries. These effects have not been considered in previous solutions but it will be found that they are not always negligible. The basic equations are non-linear but a solution will be found using a perturbation method. This method was developed for radially symmetric problems by Hodge in 1953 (6).

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

STATEMENT OF THE PROBLEM

Consider a thin annular disk rotating about its axis with an angular velocity ω radians per second. The disk is of uniform initial thickness H and initial inner and outer radii a and b. Corresponding displaced values are h, a' and b'. The problem is treated as one of generalized plane stress where the axial stress averaged across the thickness of the disk is zero.

The material is assumed to be homogeneous, isotropic, and plastically incompressible. It obeys Hooke's Law up to a well defined yield point and is thereafter perfectly plastic. It is also assumed to yield according to Tresca's yield criterion and flow according to the associated flow rule (5).

Consideration of the equilibrium of forces on an element of the disk leads to the equation

$$\frac{\partial}{\partial r}(\sigma_r h r) - \sigma_\theta h + \frac{\gamma}{g}(\omega^2 r^2 h) = 0, \tag{1}$$

where σ_r and σ_θ are the radial and tangential normal stresses, respectively. γ is the density of the disk material and r, θ, z are conventional cylindrical co-ordinates. Equation 1 holds throughout the disk.

The quantities σ_r, σ_θ, and h together with the radial displacement u are functions of r and ρ the radius of the elastoplastic boundary.

It will be found convenient to express the stresses in dimensionless form by putting,

$$\sigma = \frac{\sigma_r}{2G} \quad \text{and} \quad s = \frac{\sigma_\theta}{2G}, \tag{2}$$

where G is the modulus of elasticity in shear. In terms of these quantities Eq. 1 becomes

$$\frac{\partial}{\partial r}(\sigma h r) - sh + \frac{\gamma}{g}\frac{\omega^2 r^2 h}{2G} = 0. \tag{3}$$

Assuming the disk material to be plastically incompressible it can be shown that (7)

$$\dot{\epsilon}_r + \dot{\epsilon}_\theta + \dot{\epsilon}_z = \frac{1 - 2v}{1 + v}(\dot{\sigma} + \dot{s}), \tag{4}$$

where $\dot{\epsilon}_r$, $\dot{\epsilon}_\theta$, $\dot{\epsilon}_z$, are the principal strain rates and $\dot{\sigma}$ and $\dot{s}$ are the stress rates. Conventionally the dot indicates differentiation with respect to time. If inertia effects are neglected time may be replaced by any monotonic function of time, for example the radius of the elastoplastic boundary ρ.

The principal strain rates may be expressed as follows

$$\dot{\epsilon}_r = \frac{\partial \dot{u}}{\partial r}, \quad \dot{\epsilon}_\theta = \frac{\dot{u}}{r},$$

$$\dot{\epsilon}_z = \frac{1}{h}\frac{\partial h}{\partial \rho} = \frac{1}{h}\left[\frac{\partial h}{\partial r}\frac{dr}{d\rho} + \frac{\partial h}{\partial \rho}\right] = \frac{1}{h}\left[\frac{\partial h}{\partial r}\dot{u} + \frac{\partial h}{\partial \rho}\right].$$

Substituting these values in Eq. 4 for the strain rates there results

$$\frac{\partial h}{\partial \rho} + \frac{\partial}{\partial r}\left[h\frac{\partial u}{\partial \rho}\right] + \frac{h}{r}\frac{\partial u}{\partial \rho} = \left(\frac{1-2v}{1+v}\right)\frac{\partial}{\partial \rho}(\sigma + s). \tag{5}$$

From the accepted elastic solution it is shown that $\sigma_\theta > \sigma_r > 0$ everywhere in the elastic domain. For the present it is assumed that the same is true in the plastic domain. Therefore Tresca's yield criterion for the plastic domain may be expressed as

$$s = \mu, \tag{6}$$

where $\mu = \frac{K}{G}$ and K is the yield strength of the material in shear. If $s < \mu$ the material is elastic. The solution must satisfy Eqs. 3, 5 and 6. It must also satisfy the boundary conditions and be continuous at the elastic plastic boundary. Assuming the edges to be stress free the boundary conditions are

$$\sigma(a', \rho) = \sigma(b', \rho) = 0, \tag{7}$$

and the quantities which must be continuous at $r = \rho$ are σ, s, and u.

THE PERTURBATION METHOD

The equation of equilibrium (3) is non-linear but a solution can be obtained through the perturbation method. Each of the unknown quantities σ, s, h, ω^2 and u is expanded as a power series in terms of some convenient small parameter. It is noted that the quantity μ is of the order 0.001 for most materials and is a suitable small parameter. Thus

$$\begin{aligned}
\sigma &= \sigma_0 + \mu\sigma_1 + \mu^2\sigma_2 + \cdots \\
s &= s_0 + \mu s_1 + \mu^2 s_2 + \cdots \\
u &= u_0 + \mu u_1 + \mu^2 u_2 + \cdots \\
h &= h_0 + \mu h_1 + \mu^2 h_2 + \cdots \\
\omega^2 &= \omega_0^2 + \mu\omega_1^2 + \mu^2\omega_2^2 + \cdots.
\end{aligned} \tag{8}$$

Since σ and s are of order μ, σ_0 and s_0 can be set equal to zero. Also

$u_0 = 0$, for if μ is made zero (for rigid material) $\mu = 0$ so long as there is a containing elastic domain. Thus Eqs. 8 reduce to

$$\begin{aligned} \sigma &= \mu\sigma_1 + \mu^2\sigma_2 + \cdots \\ s &= \mu s_1 + \mu^2 s_2 + \cdots \\ u &= \mu u_1 + \mu^2 u_2 + \cdots \\ h &= h_0 + \mu h_1 + \mu^2 h_2 + \cdots \\ \omega^2 &= \omega_0^2 + \mu\omega_1^2 + \mu^2\omega_2^2 + \cdots. \end{aligned} \tag{9}$$

Equation 3 can be rearranged in the form

$$\frac{\partial}{\partial r}(\sigma h r) - sh + \phi\omega^2 r^2 h\mu = 0 \tag{10}$$

where

$$\phi = \frac{\gamma}{2gK}$$

and then on substituting for σ etc. from Eqs. 9 and collecting terms in like powers of μ the following set of equations is obtained,

$$\frac{\partial}{\partial r}(\sigma_1 r) - s_1 + \phi\omega_0^2 r^2 = 0 \tag{11a}$$

$$\frac{\partial}{\partial r}[(\sigma_2 h_0 + \sigma_1 h_1)r] - (s_2 h_0 + s_1 h_1) + \phi r^2(h_0\omega_1^2 + h_1\omega_0^2) = 0 \tag{11b}$$

etc.

Substituting the values of h and u into the compressibility Eq. 5 in a similar manner it is found that

$$\frac{\partial h_0}{\partial \rho} = 0 \tag{12a}$$

$$\frac{\partial}{\partial r}\left(h_0\frac{\partial u_1}{\partial \rho}\right) + \frac{h_0}{r}\frac{\partial u_1}{\partial \rho} + \frac{\partial h_1}{\partial \rho} = \left(\frac{1-2v}{1+v}\right)h_0(\dot{\sigma}_1 + \dot{s}_1) \tag{12b}$$

etc.

It follows that $h_0 = H$ (for h_0 is independent of ρ and $h_0 = H$ initially), and that

$$h_1 = H\left[\left(\frac{1-2v}{1+v}\right)(\sigma_1 + s_1) - \left(\frac{\partial u_1}{\partial r} + \frac{u_1}{r}\right)\right]. \tag{13}$$

In order to express the solution in terms of the original dimensions of the disk it is convenient to express the boundary conditions

$$\sigma_r(a') = \sigma_r(b') = 0$$

in terms of a Taylor's series

$$\sigma(a') = \mu\sigma_1(a') + \mu^2\sigma_2(a') + \cdots$$
$$= \mu\sigma_1(a) + \mu^2\left[\sigma_2(a) + u_1(a)\frac{\partial\sigma_1}{\partial r}(a)\right] + \cdots. \quad (14a)$$

Similarly,

$$\sigma(b') = \mu\sigma_1(b) + \mu^2\left[\sigma_2(b) + u_1(b)\frac{\partial\sigma_1}{\partial r}(b)\right] + \cdots. \quad (14b)$$

It follows that

$$\sigma_1(a) = \sigma_1(b) = 0 \quad (15a)$$

$$\left[\sigma_2(a) + u_1(a)\frac{\partial\sigma_1}{\partial r}(a)\right] = \left[\sigma_2(b) + u_1(b)\frac{\partial\sigma_1}{\partial r}(b)\right] = 0 \quad (15b)$$

etc.

THE SOLUTION

Wherever the stresses in the disk are below the yield stress they can be expressed in terms of the displacements through Hooke's Law, that is,

$$\sigma_i = \frac{1}{1-v}\left(\frac{\partial u_i}{\partial r} + v\frac{u_i}{r}\right)$$
$$s_i = \frac{1}{1-v}\left(\frac{u_i}{r} + v\frac{\partial u_i}{\partial r}\right) \quad (16)$$

where $i = 1, 2, \cdots$. Substituting appropriately in Eq. 11a, there results

$$\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r}\frac{\partial u_1}{\partial r} - \frac{u_1}{r^2} + (1-v)\phi\omega_0^2 r^2 = 0. \quad (17)$$

Equation 17 is readily integrated, the solution being

$$u_1 = A_1 r + \frac{B_1}{r} - (1-v)\frac{\phi\omega_0^2 r^3}{8}. \quad (18)$$

Substituting from Eq. 18 in Eq. 16

$$\sigma_1 = \left(\frac{1+v}{1-v}\right)A_1 - \frac{B_1}{r^2} - (3+v)\frac{\phi\omega_0^2 r^2}{8} \quad (19)$$

$$s_1 = \left(\frac{1+v}{1-v}\right)A_1 + \frac{B_1}{r^2} - (3v+1)\frac{\phi\omega_0^2 r^2}{8}. \quad (20)$$

The constants A_1 and B_1 will be evaluated through the condition that σ_1 and s_1 are continuous at the elastoplastic boundary.

The yield condition Eq. 6 may be expanded by substituting for s and then equating coefficients of powers of μ leading to the following relations

$$s_1 = 1, \quad s_2 = s_3 = \cdots = 0. \tag{21}$$

Substituting for s_1 in Eq. 11*a*, the following equation is obtained

$$\frac{\partial}{\partial r}(\sigma_1 r) - 1 - \phi\omega_0^2 r^2 = 0. \tag{22}$$

Upon integration Eq. 22 becomes

$$\sigma_1 = 1 - \frac{\phi\omega_0^2 r^2}{3} + \frac{f(\rho)}{r}. \tag{23}$$

Using boundary condition Eq. 15*a* $f(\rho)$ is evaluated and Eq. 23 becomes

$$\sigma_1 = \left(1 - \frac{a}{r}\right) - \frac{\phi\omega_0^2}{3}\left(\frac{r^3 - a^3}{r}\right). \tag{24}$$

From the condition that σ_1 and s_1 be continuous at $r = \rho$ the following values A_1 and B_1 are obtained

$$A_1 = \frac{1 - v}{1 + v}\left[1 - \frac{a}{2\rho} + \frac{\phi\omega_0^2}{12\rho}\left[(1 + 3v)\rho^3 + 2a^3\right]\right] \tag{25}$$

$$B_1 = \frac{a\rho}{2} + \frac{\phi\omega_0^2}{24}\left[(1 + 3v)\rho^4 + 4a^3\rho\right]. \tag{26}$$

In order to find the displacements in the plastic domain use is made of the flow rule associated with Tresca's yield condition. The yield condition is expressed in the form $\sigma_\theta - 2K = 0$. According to the flow rules (5) the plastic strain rate in the radial direction is proportional to $\frac{\partial}{\partial \sigma_r}(\sigma_\theta - 2K)$ and equal to zero. Therefore the strain rate in the radial direction $\frac{\partial \epsilon_r}{\partial \rho}$ is entirely elastic and can be expressed through Hooke's Law, Eq. 16, in the form

$$\frac{\partial \dot{u}}{\partial r} = \frac{1}{1 + v}(\dot{\sigma} - v\dot{\theta}) = \frac{1}{1 + v}\dot{\sigma}$$

since s is constant in the plastic region. Integrating with respect to ρ

$$\frac{\partial u}{\partial r} = \frac{1}{1+v}(\sigma + f(r)).$$

From the continuity of strain at $r = \rho$ it follows that $f(r)_{r=\rho} = -v$ and hence $f(r) \equiv -v$. Integrating again and using the condition of continuity of displacement at $r = \rho$ the plastic displacement is found. Up to this point the speed ω_0 and the radius of the elastoplastic boundary have been regarded as arbitrary. However they are not independent, and are related through the outer boundary condition, Eq. 15a.

The first term in the series for each unknown quantity is given by the following equations.

Elastic Domain

$$u_1 = \left(\frac{1-v}{1+v}\right)\left(1-\frac{a}{2\rho}\right)r + \frac{a\rho}{2r} + \frac{\phi\omega_0^2}{24}\left[2\left(\frac{1-v}{1+v}\right)\left[(1+3v)\rho^2 + \frac{2a^3}{\rho}\right]r + (1+3v)\frac{\rho^4}{r} - 4a^3\frac{\rho}{r} - 3(1-v)r^3\right]$$

$$\sigma_1 = \frac{2\rho r^2 - ar^2 - a\rho^2}{2r^2\rho} + \frac{\phi\omega_0^2}{24}\left[(1+3v)\left(2\rho^2 - \frac{\rho^4}{r^2}\right) + 4a^3\left(\frac{1}{\rho} + \frac{\rho}{r^2}\right) - 3(3+v)r^2\right] \quad (28)$$

$$s_1 = \frac{2\rho r^2 - ar^2 + a\rho^2}{2r^2\rho} + \frac{\phi\omega_0^2}{24}\left[(1+3v)\left(2\rho^2 + \frac{\rho^4}{r^2}\right) + 4a^3\left(\frac{1}{\rho} - \frac{\rho}{r^2}\right) - 3(3v+1)r^2\right]. \quad (29)$$

Plastic Domain

$$u_1 = \frac{1}{1+v}\left[(1-v)r + va + a\ln\frac{\rho}{r} + \frac{\phi\omega_0^2}{9}\left[(3v+1)\rho^3 - 3a^3\left(v + \ln\frac{\rho}{r}\right) - r^3\right]\right] \quad (30)$$

$$\sigma_1 = \left(1-\frac{a}{r}\right) - \frac{\phi\omega_0^2}{3}\left(r^2 - \frac{a^3}{r}\right) \quad (31)$$

$$s_1 = 1. \quad (32)$$

Speed

$$\omega_0^2 = \frac{12}{\phi}\left[\frac{2\rho b^2 - ab^2 - a\rho^2}{3(3+v)\rho b^4 - 4a^3(b^2+\rho^2) - (1+3v)(2\rho^2 b^2 - \rho^5)}\right]. \quad (33)$$

Two values of the speed are of particular interest

Speed for first yielding

$$\omega_0^{*2} = \frac{12}{\phi}\left[\frac{b^2 - a^2}{3(3+v)b^4 - 4a^2(b^2+a^2) - (1+3v)(2a^2b^2 - a^4)}\right]. \quad (34)$$

Speed for full yielding

$$\omega_0^{**2} = \frac{3}{\phi}\left[\frac{1}{b^2 + ab + a^2}\right]. \quad (35)$$

A second term of each series is found in exactly the same way as the first using the second equation in each set of equations derived from Hooke's Law, the equation of equilibrium, etc. The application of the method is straightforward but tedious. The equations are rather cumbersome and are not presented herein, but the numerical results for a few cases are given in Tables I and II and Figs. 1 to 4.

DISCUSSION OF RESULTS

The foregoing solution may be regarded in the following way. The first term in the series for each quantity is that which is obtained if the strains are regarded as infinitesimal and the boundary conditions satisfied in terms of the undeformed boundaries. The second term is then a correction term for the displacement of the boundaries. It will be shown that in most cases this is negligibly small so that a third term is not required.

In order to establish the importance of the second term of the solution a number of calculations have been made for the case of complete yielding, that is, when deformation is greatest. Figure 1 shows the relationship between the critical speed terms expressed as $\frac{\phi\omega_0^{**2}b^2}{12}, -\frac{\phi\omega_1^{**2}b^2}{12}$ and the diametral ratio a/b. The first term is independent of Poisson's ratio, and the second term depends on it only slightly. Figure 1 was plotted from the values in Table I.

The importance of ω_1^{**2} depends mainly on the diametral ratio, the extent of plastic flow and on the parameter μ. For small holes— $-\frac{\phi\omega_1^{**2}b^2}{12} : \frac{\phi\omega_0^{**2}b^2}{12}$ approaches 4:1. The value of μ varies between 0.001 and 0.01, the higher value being for materials with large values of the yield strength or low values of the modulus of rigidity. For low mild steels $\mu \simeq 0.001$, for high strength steels $\mu \to 0.01$ and for high strength aluminum or magnesium alloys $\mu \to 0.007$. Since $\omega^{**2} = \omega_0^{**2} + \mu\omega_1^{**2}$ the maximum effect of the second term would be to reduce the critical speed for complete yielding by about 4 per cent. In most

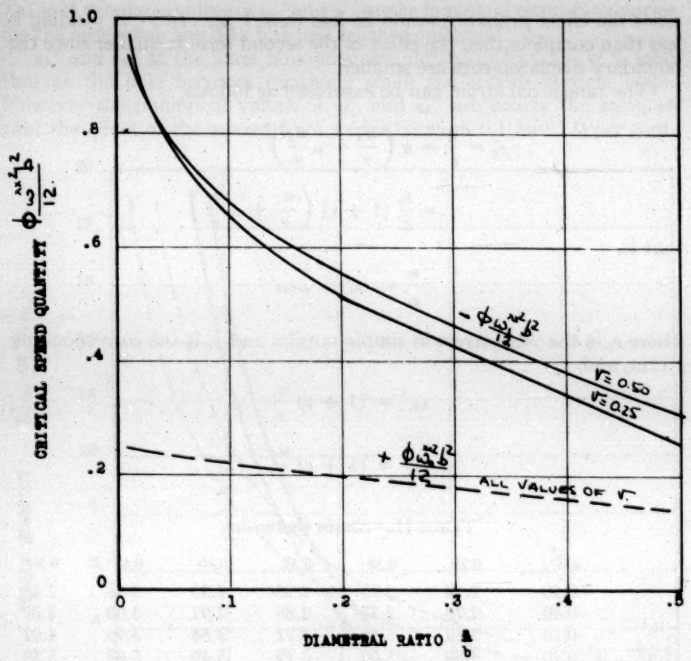

FIG. 1. Critical speed quantities for case of complete yielding.

TABLE I.—*Critical Speed Quantities.*

	a/b \ ν	0.25	0.30	0.35	0.40	0.45	0.50
	0.50	0.143					
	0.40	0.160					
$\phi\omega_0^{**2}b^2$	0.30	0.180			For all values of ν		
12	0.20	0.202					
	0.10	0.225					
	0.01	0.248					
	0.50	0.309	0.298	0.287	0.277	0.268	0.260
	0.40	0.376	0.369	0.363	0.357	0.352	0.347
$\phi\omega_1^{**2}b^2$	0.30	0.461	0.452	0.443	0.436	0.428	0.421
12	0.20	0.554	0.545	0.537	0.529	0.522	0.515
	0.10	0.676	0.670	0.664	0.658	0.653	0.648
	0.01	0.941	0.948	0.954	0.961	0.968	0.973

cases the effect probably would be less than 1 per cent. If yielding is less than complete then the effect of the second term is smaller since the boundary displacements are smaller.

The tangential strain can be expressed as follows,

$$\epsilon_\theta = \frac{u}{r} = \mu\left(\frac{u_1}{r} + \mu\frac{u_2}{r}\right)$$

$$= \frac{s_y}{E}(1+v)\left(\frac{u_1}{r} + \mu\frac{u_2}{r}\right)$$

that is,

$$\frac{\epsilon_\theta}{\epsilon_y} = \epsilon_{\theta_1}' + \mu\epsilon_{\theta_2}'$$

where s_y is the yield stress in simple tension and ϵ_y is the corresponding strain, and,

$$\epsilon_{\theta_1}' = (1+v)\frac{u_1}{r}$$

$$\epsilon_{\theta_2}' = (1+v)\frac{u_2}{r}.$$

TABLE II.—*Strain Quantities.*

	a/b \ v	0.25	0.30	0.35	0.40	0.45	0.50
	0.50	2.18	2.23	2.28	2.33	2.38	2.43
	0.40	2.70	2.77	2.85	2.92	3.00	3.07
$\epsilon_{\theta_1}{}^1$	0.30	3.49	3.60	3.72	3.84	3.96	4.07
$\rho=b$	0.20	4.89	5.09	5.29	5.49	5.69	5.89
$r=a$	0.10	8.53	8.98	9.43	9.89	10.33	10.78
	0.01	63.34	68.29	73.23	78.18	83.12	88.08
	0.50	0.67	0.69	0.71	0.73	0.76	0.78
	0.40	1.09	1.11	1.14	1.17	1.19	1.22
$\epsilon_{\theta_2}{}^1$	0.30	1.48	1.56	1.65	1.73	1.82	1.86
$\rho=b$	0.20	4.39	4.56	4.74	4.92	5.10	5.29
$r=a$	0.10	13.35	13.85	14.35	14.88	15.42	15.96
	0.01	471.6	499.6	526.8	553.1	576.5	603.4
	0.50	0.080	0.094	0.108	0.122	0.135	0.148
	0.40	0.080	0.097	0.111	0.125	0.138	0.151
$\epsilon_{\theta_3}{}^1$	0.30	0.081	0.098	0.117	0.135	0.154	0.173
$\rho=r=b$	0.20	0.092	0.116	0.142	0.169	0.198	0.226
	0.10	0.110	0.147	0.177	0.214	0.254	0.295
	0.01	0.234	0.305	0.382	0.464	0.551	0.642
$\epsilon_{\theta_4}{}^1$ $\rho=r=b$		1.00 for all values of a/b and all values of v					

Table II tabulates values of ϵ_{θ_1}' and ϵ_{θ_2}' at the inner and outer boundaries for the case when the disk has just reached full yield.

ϵ_{θ_1}' and ϵ_{θ_2}' at the inner boundary are plotted in Fig. 2. It is seen that as the hole becomes relatively small the strains become large. However the numerical values of ϵ_{θ_1}' and ϵ_{θ_2}' are nearly the same so that the effect of the second term varies between 0.1 and 1.0 per cent.

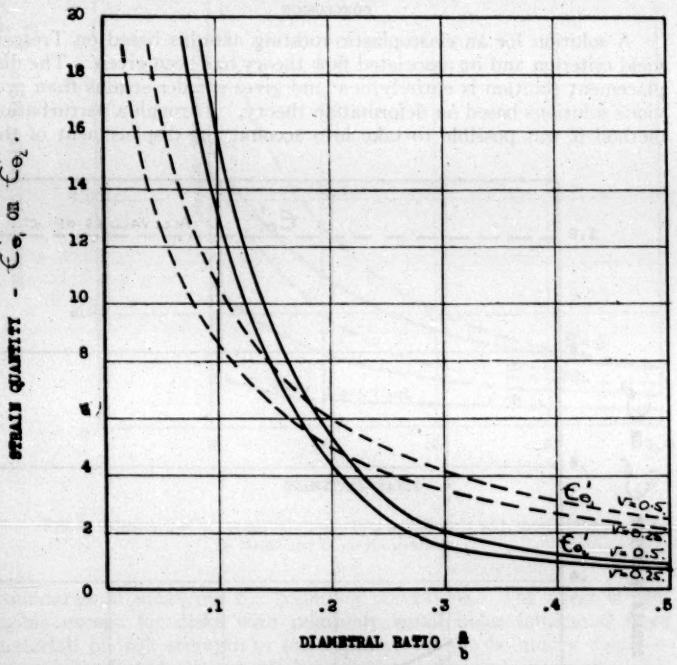

FIG. 2. Tangential strain quantities at inner boundary for case of complete yielding.

Figure 3 shows ϵ_{θ_1}' and ϵ_{θ_2}' at the outer boundary for the case of full yield. ϵ_{θ_2}' remains small in comparison with ϵ_{θ_1}' so that the effect of the second term can be disregarded.

The first terms of the stress and speed solutions in terms of ρ are identical with the Nadai and Donnell solution. This is so because the first term of the stress solution is obtained without any reference to the plastic stress-strain relation. However, the displacement solution is entirely different. The quantity $\frac{\epsilon_\theta}{\epsilon_\nu}$ has been calculated for the same

example used by Nadai and Donnell. The example used takes $v = 0.3$ and $\gamma = 0.28$. Three cases are considered, $\rho = a$, $\rho = 0.6b$ and $\rho = b$. Figure 4 compares the values given by the two solutions. Nadai and Donnell used $\frac{\epsilon_\theta}{\epsilon_y} = 1$ at $r = \rho$ as a starting point for numerical integration. This is only correct for $\rho = a$ or $\rho = b$.

CONCLUSION

A solution for an elastoplastic rotating annulus based on Tresca's yield criterion and its associated flow theory has been given. The displacement solution is entirely new and gives smaller strains than previous solutions based on deformation theory. Through a perturbation method it was possible to take into account the displacement of the

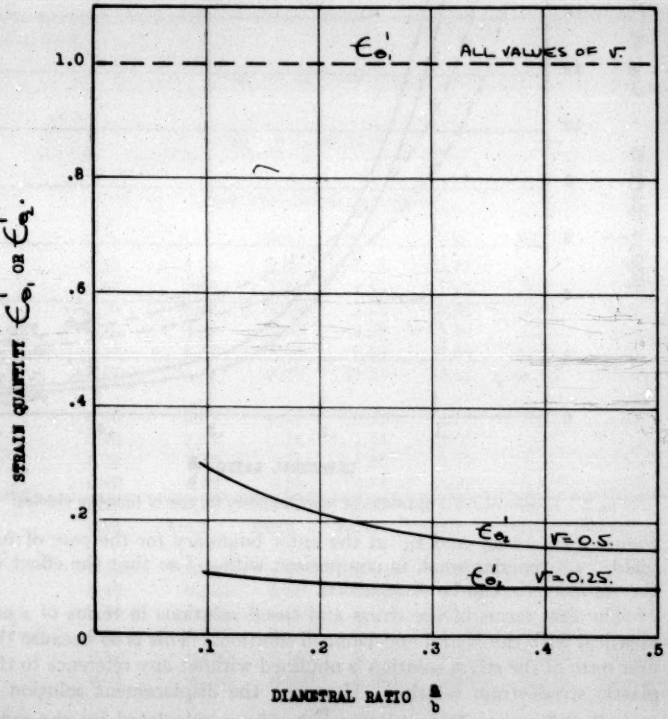

FIG. 3. Tangential strain quantities at outer boundary for case of complete yielding.

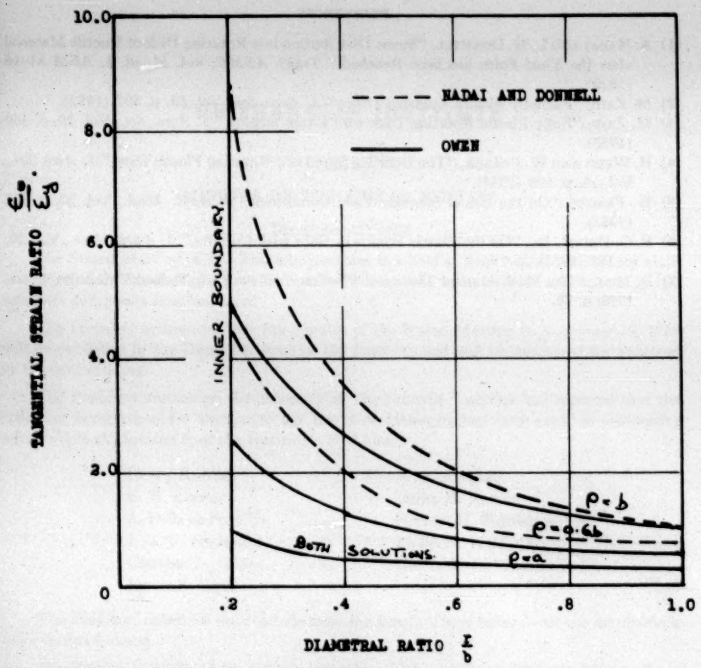

FIG. 4. Comparison of tangential strain ratios at full yield with those obtained by Nadai and Donnell's method.

boundaries in satisfying the boundary conditions. The effect is negligible except for disks with relatively small holes fabricated from materials of high strength or low rigidity. If the boundary displacement is neglected the use of the associated flow rule permits a particularly simple displacement solution, namely that given by the quantities u_1.

Acknowledgments

The author is indebted to Dr. L. W. Hu, who suggested this problem, for his advice and criticism. The work was carried out as part of a project "Theoretical and Experimental Studies on the Effects of High Pressures on the Mechanical Properties of Metals" sponsored by the Office of Ordnance Research, U. S. Army.

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THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

December 17, 1958

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Mr. Wynn Laurence LePage, President, called the meeting to order with approximately 160 members and guests in attendance.

The President announced that the minutes of the Stated Meeting of November 19, 1958 will be published in the December issue of the JOURNAL and will be submitted for approval at the next meeting.

The President announced the Secretary of The Franklin Institute had reported that the following nominations for election to our Board of Managers had been made in conformity with Article IV, Section 5, of the Institute's By-Laws:

Henry B. Allen	R. G. Rincliffe
G. H. Clamer	James H. Robins
A. Felix duPont, Jr.	Herbert H. Swinburne
J. G. R. Heckscher	Philip H. Ward, Jr.
Clarence L. Jordan	Harleston R. Wood
Lionel F. Levy	

The President called for nominations from the floor. There being none, the nominations were declared closed.

The President appointed the following members of the Institute, who are neither officers nor nominees, to act as Tellers of the Election: Dr. George S. Crampton, Dr. Joseph S. Hepburn, and Mr. Howard Stoertz.

In introducing the speaker of the evening, Dr. J. Allen Hynek, who is presently Associate Director of the Smithsonian Astrophysical Observatory, and previously Astronomer at the Perkins Observatory, Professor of Astronomy and Dean of the Graduate School, Ohio State University, the President stated that Dr. Hynek heads the visual and satellite tracking programs for the United States, working with the U. S. National Committee for the International Geophysical Year.

Dr. Hynek's talk on "Stopping Satellites in their Tracks" dealt with the equipment required and the geographical locations that made it possible to obtain precise photographic tracking of satellites, and he illustrated through slides and models the amazing degree of accuracy which is attainable through these photographic telescopes. He spoke with enthusiasm of the Moonwatch Visual System set up throughout the country by Armand Spitz, former Director of Museum Education at The Franklin Institute. Dr. Hynek's talk was received with interest and was followed by a full discussion period.

The meeting was adjourned by the President at 10:00 o'clock.

WILLIAM F. JACKSON, JR.
Secretary

MEMBERSHIP

GOLD MEMBER FOR 1958 HONORED

On November 19, 1958, the Institute paid tribute to Mr. William M. C. Kimber, who had completed fifty years of continuous membership during 1958. During the dinner which preceded the regular Institute meeting, Mr. Kimber was introduced by Mr. Wynn Laurence LePage, President. At the Stated Meeting, Mr. Morton Gibbons-Neff, Chairman of the Membership Committee, presented Mr. Kimber to Mr. LePage and to his fellow members. Mr. LePage then gave Mr. Kimber a gold key and gold membership card, symbolizing his fifty years of membership in the Institute.

BIOGRAPHICAL SKETCH OF WILLIAM M. C. KIMBER

Born on July 16, 1882, in Germantown, William M. C. Kimber was educated at Germantown Friends School and Haverford College. In 1904 he became an apprentice with the Link Belt Company working with Fred Taylor in his development of the Taylor System of Scientific Management.

In 1905 Mr. Kimber joined Leeds and Northrup for which firm he later developed a modified Taylor System. In 1924 he resigned as Vice-President in charge of production to organize a company for the manufacture of the SWORD Domestic Oil Burner. Production of the oil burner was discontinued in 1930. Mr. Kimber then worked for six years as an engineer in New York City. From 1937 through 1942 he was employed by Philco Corporation. In 1943 he became the business manager of Shipley School in Bryn Mawr. Five years later he went with the Fox Products Company, for which he still works.

During his career, Mr. Kimber has belonged to the Society of Industrial Engineers, Taylor Society, Society of Electrical Engineers and the American Metal Manufacturers Association.

Mr. Kimber and his wife, Elizabeth, live in Philadelphia.

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William M. C. Kimber, 1958 Gold Member, signs the distinguished visitors book, under the supervision of two other Gold Members, Dr. G. H. Clamer (1941) and Mr. James M. Caird (1951).

NEW MEMBERS

The following elections were approved on October 2, 1958 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

John S. Adractas
John Lee Baringer
C. R. Billman
Thomas E. Boyd
Walter H. Caldwell
D. E. Childs
Edward J. Connelly
Robert J. Curry, Jr.
E. S. Daugherty
R. W. Dixon
T. I. Eldridge
Jack Elgart
James S. Evans
August Frankena
M. Michael Garber
A. Stuard Graham
Forrest L. Grimmett
Dwight J. Harris
Jerome G. Harrison
John F. Heberer
Frederick J. Heinze
Carl Helmetag, Jr.
William D. Hopkins
Walter C. Huffman
Charles B. Hugins
Louis E. Huhman
Irving Hyams
Harry J. Irrgang, Jr.
Clifford B. Ives
Arthur W. Jacobs
Burke W. Jay
David G. Jensen
J. N. Junkins
Clifford R. Koelle
Marquis A. McClellan
F. M. McLarty
Mrs. Grenville Montgomery
Lionel E. Moskowitz
W. J. Novak
Hilliard W. Paige
Warren L. Phillips
Ivan E. Robinson
Bernard Sabreen
Sidney S. Shapiro
Lloyd W. Spangler, Jr.
Tibor Tompa
Harleston R. Wood
John J. Wrenn
Harry M. Yarrow

EDUCATOR

LeRoy A. Brothers
Peter Zilahy Ingerman
Harold F. Martin
William H. Parsons
Louis Schwartz

NECROLOGY

Israel Abrams '47
William M. Boyle '51
A. C. Carlton '47
Kern Dodge '03
Joseph Esherick '36
R. Elmer Peoples '46
Beatty H. Rhinesmith '51
Lewis G. Schlehner '32

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, December 10, 1958.)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 10, 1958.

MR. JOSEPH GRAY JACKSON in the Chair.

The following report was presented for final action:

No. 3224: Work of Irving Wolff.

This report recommended the award of an Elliott Cresson Medal to Irving Wolff, of Princeton, New Jersey, "In consideration of his many important contributions to the science of electronics as evidenced by his productiveness over a period of many years in many diverse fields such as acoustics, optics, radio, infra-red detection and radio frequency heating and especially in view of his pioneering work in the centimeter wave field and his contributions to microwave radar development."

D. S. FAHRNEY,
Secretary to Committee

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 362 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

RECENT ADDITIONS

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PROCEEDINGS OF RECENT SYMPOSIA HELD AT THE FRANKLIN INSTITUTE[1]

"Cleaner Air for Urban Areas," held October, 1956, 69 pp., illus., $3.

"Particulate Emission," held October, 1957, 105 pp., illus., $3.

"The Airways Modernization Board—Its Mission and Methods," held December, 1957, 160 pp., illus., $4.

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[1] These publications are available, on order to The Franklin Institute.

BOOK REVIEWS

SWITCHING CIRCUITS AND LOGICAL DESIGN, by Samuel H. Caldwell. 686 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $14.00.

For many years, the most extensive and complex assemblages of switching circuits were probably the dial telephone systems. Other important applications were made in the traffic regulating systems used by railroads and rapid transit systems, and in the punched card computing systems developed before the advent of large scale digital computing machines. Logical design methods were developed for various specific applications and used with good results.

The new problems presented in the design of large scale digital computing systems, and in the design of large and complex automatic control systems for manufacturing operations, have emphasized the need for a more thorough understanding of the basic principles of the subject. They have pointed up the need for a more comprehensive treatment of switching circuits and logical design, with extensions into areas such as sequential circuits, which have now become important problems, and which have not been adequately covered in earlier over-all treatments of the subject.

While many able men have worked intensively and productively in this field in the past dozen years, their work has been published principally in the form of isolated papers dealing with specific phases of this subject. Dr. Caldwell is the first to present a thorough and comprehensive treatment of the subject, using the most up-to-date techniques, in a form which can be of real and immediate value to engineers working in this field. It should be noted, however, that although this text possesses this very direct sort of utility, it leaves the reader with a sense that there are vast unexplored areas waiting for him to develop and stimulates an urge to extend and amplify the material that has been presented to him.

After pointing out some of the many applications for switching circuits and logical design and describing typical components of switching systems, Dr. Caldwell proceeds with an exposition of the algebraic ideas of George Boole that are applicable to the processes of switching circuit analysis and synthesis. The necessary theorems are developed in a simple and consistent manner, using only plus signs to indicate logical addition, and dots to indicate logical multiplication, instead of the cup and cap symbols frequently used in treatments of this algebra. He demonstrates that this algebra constitutes a sound logical system, which does not require physical support, before proceeding with its physical interpretation in terms of switching circuits. In the course of this interpretation, the effectiveness of the algebra for describing physical systems is convincingly demonstrated.

This leads naturally into the analysis of series-parallel contact networks. The use of the table of combinations for synthesis is discussed, and algebraic methods of circuit simplification are demonstrated. The concepts of cut and tie sets and complementary networks are introduced.

The chapter on minimization methods introduces some of the procedures, graphical aids, and algorithms that have been found useful in reducing algebraic expressions to optimum forms, and that also may show the existence of alternate solutions. Use of numerical descriptions of switching functions and graphical methods of representation of these functions are discussed. It is shown that these can lead to the construction of the Karnaugh Map, which is extremely useful in operating on four-variable functions, and methods of minimization based on the Karnaugh Map are covered in some detail. This is followed by a discussion of the Quine-McCluskey method of minimization of functions of five or more variables.

Chapters are devoted to multi-terminal contact networks, symmetric functions, and the synthesis of non-series-parallel networks. Examples of the application of the methods developed above to electronic and solid state devices in combinational switching circuits are also discussed.

A chapter is devoted to the discussion of the switching aspects of codes, with a discussion

of error detection, and error detection and correction methods. Another chapter discusses the switching properties of iterative networks, analyzes their structure, and describes design and synthesis methods for circuits of this class.

The synthesis of sequential switching circuits is given the emphasis that is justified by the current importance of this subject. The methods are based on the work of D. A. Huffman. It is shown that the Karnaugh Map can be developed to indicate the stability of the various states of the system, and the manner in which the stable state must be reached from each of the unstable states. Race conditions can then be identified readily.

One of the modifications of the Karnaugh Map can be used to construct a primitive flow table. In certain respects, this table bears the same relation to sequential problems that the table of combinations bears to combinational problems. The primitive flow table may be reduced by merging rows which satisfy certain specified conditions. From this we may derive an excitation matrix, from which a circuit free from critical race conditions may be synthesized. Methods of preventing and removing hazards are discussed.

In the process of transition from the primitive flow diagram to the excitation matrix, it is necessary to assign a set of secondary states to represent the rows in the reduced flow tables. In many problems, this procedure may not be simple or straightforward, and methods for dealing with these cases are discussed in the chapter on secondary assignment methods.

This is followed by a chapter on applications of electronic and solid state elements in sequential circuits. The body of the text closes with a chapter on pulsed sequential circuits. Here, several of the problems encountered in digital computer design are analyzed and discussed, using the basic techniques developed above. Five appendices covering the essentials of some of the mathematical techniques used in the text complete the volume.

Throughout, the author has drawn upon his extensive teaching experience to make the text both rigorous and lucid, and to place emphasis on a sound understanding of principle and methods. Application of these methods to specific problems will not be difficult for a thoughtful reader of this text.

LEWIS P. TABOR
The Franklin Institute Laboratories

PHYSICS AND CHEMISTRY OF THE EARTH, VOL. II, edited by L. H. Ahrens, Frank Press, Kalervo Rankama and S. K. Runcorn. 259 pages, plates, diagrams, 6 × 9¼ in. New York, London and Paris, Pergamon Press Inc., 1957. Price, $10.00.

The wide geographic distribution of the four editors is a clue to the broad international and universal character of this book. The seven chapters cover widely divergent but related physical and chemical properties of the Earth and the universe.

Chapter 1 is a study of the movement and behavior of bodies of water and seeks to establish relationships between observed phenomena produced through use of models. Bodies of water are divided roughly into three categories displaying local circulation, regional circulation and planetary circulation. Details of model construction are discussed. This study should be useful reference material for hydraulic engineers and students of oceanography.

Chapter 2 reviews analytical geochemical data from the late 1800's to the present. These data are examined from the standpoint of both quantity and quality. The significance of analytical error is evaluated. This study should be of value to serious students of geology from the standpoint of choosing the approach to analytical problems. It is not a manual of procedure.

In Chapter 3 an attempt is made to set reasonable boundaries within which theories of the origin of the solar system may be accepted as reasonable. Evidence is presented supporting the theory that the terrestrial planets were formed at low or moderate temperatures. The review of differences in proportions of elements in the major planets and stars will be of interest to all students of the solar system.

Chemical composition of sea water and the ocean floor is discussed in Chapter 4. An attempt is made to correlate the composition of sea water and the composition of animal debris on the ocean floor. This is followed in Chapter 5 by discussion of the methods of geophysical exploration of the sea. Various methods for determining water depth and the physical characteristics of the ocean floor are discussed.

In Chapter 6 the history, chemistry and cosmic abundance of gallium, indium and thal-

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lium are presented together with a thorough review of the literature on these rare elements.

The final chapter gives observational and mathematical analyses of the motion of the Earth's pole with some theories on the physical structure of the Earth.

Anyone interested in the physical and chemical composition of the universe will find this book of interest.

R. J. EMMONS
Philadelphia Quartz Co.

NATURAL AERODYNAMICS, by R. S. Scorer. 312 pages, diagrams, plates, 5½ × 8½ in. New York, London & Paris, Pergamon Press Inc., 1958. Price, $9.00.

The title of this book sounds imposing and it is, but regardless of the complexity of the subject it can be read and enjoyed by anyone with a little scientific training. The book is derived from lectures given by the author to audiences composed of aviators, engineers, geographers, meteorologists, mathematicians and other interested persons.

The basic concepts involved in air motion in nature are developed with many examples and many excellent photographs and drawings. The author has tried to show how one can argue what it is necessary to specify in order that there shall exist solutions to the problems posed and to explain qualitatively what the answers are, rather than duplicate the mathematical treatment which may be found in other books.

The chapter headings give some idea of the subject matter: inertia forces, motion on a rotating earth, vorticity, viscosity, boundary layers, wakes and turbulence, buoyant convection, plumes and jets, air waves, clouds and fallout and philosophy of method. The delightful digressions from the main subject cannot be indicated quite so easily but these in my opinion make the book more readable. As an example in the discussion of buoyant convection there is a section on natural exploitation of atmospheric thermals. This described birds that make use of thermals to reduce the effort required to remain airborne, dynamic soaring of albatrosses, the habits of dragonflies and aphids, the migration of locusts and even some comments about fishes.

The chapter on air waves is one of the most interesting for a general reader. It covers explosion waves, classified as little bangs, big bangs and aeroclysms, tidal waves, mountain waves, waves produced by thermals and again a diversion on noise in nature—why do leaves on trees dance, flags flap, swan's wings whine and wires sing in the wind.

For those interested in air pollution there is much pertinent material particularly the chapter on plumes and jets which contains a discussion of Sutton's theory. One paragraph which should be considered by many involved in air pollution is here quoted.

> The contemporary concern about the dispersion of effluents stems from the disastrous consequences of air pollution mostly in modern cities. The greatest disasters occur under weather conditions which depart most seriously from the average in such a way as to invalidate formulae designed to apply to average conditions. Our social consciences cannot be salved by scientific studies, however elegant, of dispersion of pollution in average weather. We must study especially those weather conditions which render the pollution noxious, learn to understand the processes which then operate, and plan our towns accordingly. The behaviour of the atmosphere is so varied (and for this we should be grateful for life is thereby enriched) that no manageable theory can possibly treat all its behaviour. If we are to treat some aspects rather than others, then we see that the consequences of pollution require that special, rather than average, conditions shall receive immediate attention. Two final remarks: average conditions in a medium of such varied behaviour as the atmosphere are practically a fiction, as rare as most of the special conditions which concern us as social beings: but he is a poor scientist who in his ivory tower heeds not special human problems if they are relevant to his discipline.

The closing chapter on how to approach the problems of Natural Aerodynamics not only discusses various methods but gives us a look at the author's philosophy.

In addition to an index there is a list of problems and experiments for discussion. This book is highly recommended for the non-mathematician, the non-meteorologist and any others who feel they are non-specialists.

W. E. SCOTT
The Franklin Institute Laboratories

NUCLEAR MASSES AND THEIR DETERMINATION, edited by H. Hintenberger. 267 pages, illustrations, 5½ × 8½ in. New York and London, Pergamon Press Inc., 1957. Price, $14.00.

This book constitutes the proceedings of the conference held in the "Max Planck Institute für Chemie" Mainz, July 10 to 12, 1956. In

honor of the sixtieth birthday of J. Mattauch, its purpose was to bring together in one meeting those who have determined nuclear masses by mass spectroscopy and those who have determined them from nuclear reactions, thereby contributing to the effort to iron out some of the discrepancies which still exist in their results.

Following a survey, by K. T. Bainbridge, of the present status of the determination of nuclear masses, the papers are grouped under the following general headings: Atomic Masses and Nuclear Structure, Mass-Spectroscopic Results, Nuclear Masses from Nuclear Reactions, Comparison Between Nuclear Masses from Q Values and Mass-Spectroscopic Doublets, Atomic Masses from Microwave Spectra (one paper), Instruments, and Miscellaneous.

As may be expected in a symposium on this subject, the precision of measurements is a recurring theme. There were twenty-six papers in all, nearly half from the United States, two of them from The Bartol Research Foundation. Of the twenty-six, eight are devoted to the Instruments section, and all of these deal with improvements in mass spectroscopy.

Some papers dealt with the latest experimental results, often of a preliminary nature. In others, the general status of the data was considered. One of these, by J. Mattauch, was concerned with the least-squares adjustment of the data obtained by mass spectroscopy and from Q values to arrive at a consistent system of mass values for light nuclei. As indicated by the general headings, the papers covered a wide range of closely related subjects.

This volume is a valuable contribution to the literature of the progress of the work of obtaining nuclear mass values of ever increasing precision.

G. P. WACHTELL
The Franklin Institute Laboratories

AN INTRODUCTION TO THE DYNAMICS OF AIRPLANES, by H. Norman Abramson. 225 pages, diagrams, 6 × 9 in. New York, The Ronald Press Co., 1958. Price, $4.50.

The introductory chapters I, II and III cover a brief review of basic principles in vibration analysis, as a background for the discussion of wing flutter, problems in aeroelasticity, with impulse loading and flight stability. The formulation of the flutter vibration of a wing is reduced to an equivalent wing mass and elastic constants, representing a portion of the wing of unit span to a two-degree-of-freedom dynamical system, with coupling of bending and torsion. The complicated Theodorsen analysis for the applied aerodynamic reactions are quoted. At this point it would seem possible to substitute some simplifying analysis representing the essential characteristics of the physical phenomena of the aerodynamic loading in view of the simplified treatment of an otherwise more complicated three dimensional wing problem. The use of normal modes and corresponding orthogonal functions of the over-all elastic structures, as discussed in Chapter VII and the latter part of Chapter V, form a clear background for various applications in aero-elasticity, such as the dynamic response in elastic systems. The author covers interesting chapters on impulsive loading problems and flight stability as well as experimental and analog methods, including other miscellaneous topics on testing with models.

On the whole, this work is a very useful and matured technical survey of basic problems relating to the dynamics of airplanes.

RUPEN EKSERGIAN
The Franklin Institute Laboratories

GAS DYNAMICS, by Ali Bulent Cambel and Burgess H. Jennings. 415 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $11.00.

Gas Dynamics has an admirable goal, which is to serve both as a readable text book for advanced undergraduate and beginning graduate students and as a ready reference for those already beyond the formal halls of learning. The intent is commendable and the authors have achieved their difficult goal with reasonable success.

Basic definitions and concepts pertinent to a study of fluid flow are presented in the first two chapters. With that foundation the authors then develop the fundamental equations of fluid flow both with and without friction and with and without heat transfer. From the point of view of the mechanical engineer Chapter Six—Wave Phenomena—is excellently presented and amply illustrated. The treatment of shock waves, both strong and weak is reasonably complete and should stimulate the reader to further study.

Thermochemistry and combustion are

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treated in Chapters 8 and 9; the treatment of combustion is all too brief. However, the authors are to be commended for a well presented introduction to combustion phenomena and for an excellent group of references for continued study. Chapter 10 is an introduction to multidimensional flow and may require the reader to brush up on his vector notation and vector calculus. This reviewer was pleased to see the method of characteristics utilized in simple nozzle design problem. The utilization of developed methods is all too often omitted from text books and should form an integral part of the presentation.

The chapter on experimental techniques and measurements presents to the reader a good description of methods used in shock tubes and wind tunnels to analyze flow around bodies. Unfortunately, some of the illustrations are poor examples of the method being described. Flow direction is not noted and the illumination is not always adequate to emphasize the point under discussion.

The interaction between aerodynamics, thermodynamics, and chemistry is described in the final chapter. The discussion as the authors state ". . . is not planned as a profound analysis of this most complex subject." However, it is a well presented invitation to further study.

One final comment of an economic nature is warranted. As mentioned earlier, the book is intended primarily as a text for advanced undergraduates and Masters' candidates. However, this reviewer is somewhat hesitant to recommend the book as a text because of its cost. At least one-half the contents of *Gas Dynamics* has been served to the advanced undergraduate and beginning graduate in his required intermediate courses through texts far less costly and equally or more detailed than the book under review. By remaining true to their title the authors could have presented a detailed treatment of aerothermodynamics and have made the presentation much more economically palatable to the student.

ELLIOTT L. KATZ
The Martin Company

PUBLICATIONS RECEIVED

THEORETICAL MECHANICS. AN INTRODUCTION TO MATHEMATICAL PHYSICS, by Joseph Sweetman Ames and Francis D. Murnaghan. Unaltered and unabridged republication of the first edition. 462 pages, diagrams, $5\frac{1}{4} \times 8$ in. New York, Dover Publications, 1958. Price, $2.00 (paper).

THE FUNDAMENTAL PRINCIPLES OF QUANTUM MECHANICS WITH ELEMENTARY APPLICATIONS, by Edwin C. Kemble. Unabridged and corrected republication of the first edition. 611 pages, $5\frac{1}{2} \times 8\frac{1}{4}$ in. New York, Dover Publications, 1958. Price, $2.95 (paper).

STATICS AND THE DYNAMICS OF A PARTICLE, by William Duncan MacMillan. Unabridged and unaltered republication of the first edition. 430 pages, diagrams, $5\frac{1}{4} \times 8$ in. New York, Dover Publications, 1958. Price, $2.00 (paper).

VECTOR ANALYSIS WITH AN INTRODUCTION TO TENSOR ANALYSIS, by A. P. Wills. Unabridged and corrected republication of the first edition. 285 pages, $5\frac{1}{2} \times 8$ in. New York, Dover Publications, 1958. Price, $1.75 (paper).

AN INTRODUCTION TO FOURIER METHODS AND THE LAPLACE TRANSFORMATION, by Philip Franklin. Unabridged and corrected edition of the work first published in 1949 under the title "Fourier Methods." 289 pages, diagrams, $5\frac{1}{4} \times 8$ in. New York, Dover Publications, 1958. Price, $1.75 (paper).

INTRODUCTION TO BESSEL FUNCTIONS, by Frank Bowman. Unabridged and unaltered republication of the first edition. 135 pages, $5\frac{1}{4} \times 8$ in. New York, Dover Publications, 1958. Price, $1.35 (paper).

PHILOSOPHY OF ATOMIC PHYSICS, by Joseph Mudry. 136 pages, $5\frac{1}{4} \times 8\frac{1}{4}$ in. New York, Philosophical Library, 1958. Price, $3.75.

PROCEEDINGS OF THE EIA CONFERENCE ON MAINTAINABILITY OF ELECTRONIC EQUIPMENT, sponsored by the Engineering Department, Electronic Industries Association. 104 pages, illustrations, $8\frac{1}{2} \times 11$ in. New York, Engineering Publishers, 1958. Price, $5.00 (paper). (Sole distributor outside the U. S. A.: Interscience Publishers.)

FUNDAMENTALS OF PIPE DRAFTING, by Charles H. Thompson. 66 pages, illustrations, $8\frac{1}{2} \times 11$ in. New York, John Wiley & Sons, Inc., 1958. Price, $3.50 (paper).

BOOK NOTES

ADVANCES IN GEOPHYSICS, VOL. 5, edited by H. E. Landsberg and J. Mieghem. 325 pages, drawings, 6 × 9 in. New York, Academic Press, Inc., 1958. Price, $10.00.

The fifth volume in this series contains six papers on widely varying topics. Each paper has been prepared by an expert, and each begins with a very detailed table of contents and concludes with a voluminous set of references. The paper "From Polar Years to IGY," by N. C. Gerson, gives a comprehensive history of man's periodic, concentrated studies of geophysics, on an international scale, including the current IGY program. The editor, in his Foreword, calls attention to the importance of evaluating the data gathered during IGY, and opens future volumes of the series to articles stemming from such data.

The remaining survey articles in Vol. 5 are: "Microseisms," by B. Gutenberg (who has been doing research in this field for 50 years); "The Size and Shape of the Earth," by R. A. Hirvonen, who discusses triangulation, astronautical coordinates, spirit leveling and gravity measurements; "Oceanic Tides," by A. T. Doodson; "Ultraviolet Absorption Processes in the Upper Atmosphere," by K. Watanabe (with 253 references); and "Physics of Cloud Modification," by James E. McDonald, who advocates more intensive studies of cloud seeding techniques to augment our limited water supply.

PROGRESS IN SEMICONDUCTORS, VOL. 3, edited by A. F. Gibson, P. Aigrain and R. E. Burgess. 210 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $8.50.

Volume 3 in this annual series, designed to bring the specialist abreast of developments in fields related to his own, covers a wider range than the previous volumes, while maintaining the high standards set earlier. The seven papers are: "The Magnetoresistivity of Germanium and Silicon" by M. Glicksman; "The Chemical Purification of Germanium and Silicon" by J. M. Wilson; "Electronic Conductivity of Silver Halide Crystals" by J. W. Mitchell; "Silicon Junction Diodes" by D. E. Mason and D. F. Taylor; "Lifetime of Excess Carriers in Semiconductors" by A. Many and R. Bray; "Scattering and Drift Mobility of Carriers in Germanium" by M. S. Sodha; and "Electronic Processes in Cadmium Sulphide" by J. Lambe and C. C. Klick.

EPOXY RESINS, by Irving Skeist. 293 pages, illustrations, 5 × 7½ in. New York, Reinhold Publishing Corp., 1958. Price, $5.50.

Epoxy Resins is the eighth of Reinhold's Plastics Applications Series. Semi-technical in nature, they are aimed at design engineers, manufacturers, raw-material suppliers, purchasing agents and students. The epoxies are treated in this volume covering their uses as casting and potting compounds, adhesives, and plasticizers. Trade names and manufacturing procedures are dealt with and there is a special section on epoxy coatings. A list of 298 references to periodical literature is a valuable aid to the technical reader seeking additional information.

GUM PLASTICS (RUBBER-MODIFIED PLASTICS), by M. S. Thompson. 193 pages, illustrations, 5 × 7½ in. New York, Reinhold Publishing Corp., 1958. Price, $4.50.

Three principal types of the rubber-modified plastic resins are treated in this ninth book of the Plastics Applications Series—impact-polystyrene polymers, acrylonitrile-butadiene-styrene polymers, and impact-rigid polyvinyl chloride polymers. The author devotes 38 pages to the general properties (toughness and mechanical strength), 20 pages to the basic chemistry and manufacture of gum plastics, 40 pages to processing and fabrication (injection and extrusion molding, sheet forming, calendering, machining, cementing and painting), and about 75 pages to applications. Clear diagrams and interesting photographs add to the presentation.

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TRANSFORM METHOD IN LINEAR SYSTEM ANALYSIS, by John A. Aseltine. 300 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.50.

A senior or graduate level text, this book concerns the application of integral transforms to the analysis of physical systems described by linear differential equations. The author uses the transform method to explain and to solve engineering problems in electrical network analysis, in mechanical systems and in systems with random inputs (both direct and frequency descriptions). Discussed in detail are Laplace transforms, Fourier series, Fourier transforms, z-transforms and (briefly) Mellin transforms. The author emphasizes the unified approach to linear systems (electrical, mechanical, thermal) through use of the transform method, and shows how the transform method can give an understanding of physical phenomena.

THE JUNCTION TRANSISTOR, edited by E. Wolfendale. 394 pages, diagrams, 6 × 9¼ in. New York, The Macmillan Company, 1958. Price, $7.50.

Written by engineers and physicists of the Mullard Radio Valve Co., Ltd., this book covers the physics of transistors, the design of transistor circuits and their applications. Aimed at physicists, engineers and students, the book will be of most use to designers of all types of circuits. The various chapters treat the characteristics of junction transistors, four-terminal networks and the transistor equivalent circuit, direct current biasing and audiofrequency amplification, class C amplification, sinusoidal oscillators, amplitude modulation and demodulation, the junction transistor in nonlinear circuits, and transistor d.c. converters. There is an appendix on transistor measurements.

SAMPLED-DATA CONTROL SYSTEMS, by John R. Ragazzini and Gene F. Franklin. 331 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $9.50.

Intended as a graduate text or a self-study text for practicing engineers, this new book brings together much material previously found only in scattered sources. It deals primarily with the theory of sampled-data systems, which can be used in control and communications systems. The authors have assumed that the reader is well versed in differential equations, the Laplace transformation, linear feedback control theory, and have some knowledge of probability and statistics. The z-transform method is used extensively and methods for the time domain synthesis of single-rate and multi-rate sampled-data systems are presented (these latter were developed under the direction of Dr. Ragazzini). Design and analysis methods are illustrated by completely worked-out examples.

CONTRIBUTIONS IN GEOPHYSICS. VOLUME I ON EARTH SCIENCES, edited by Hugo Benioff, Maurice Ewing, B. J. Howell, Jr. and Frank Press. 244 pages, diagrams, 5½ × 8½ in. New York and London, Pergamon Press Inc., 1958. Price, $9.00.

This first volume of a new series on Earth Sciences honors Beno Gutenberg for his contributions to geophysics during a lifetime of teaching and research. The authors are the world's outstanding geophysicists. Seventeen papers (one in French and one in German) represent work from twelve countries and deal authoritatively with all phases of geophysical research. They include studies of seismic body waves, energy in earthquakes, seismic activities in Japan, free oscillations of the Earth, gravity formulas, data-processing in geophysics, geomagnetic drift, etc. The volume is a scholarly testimonial to the man who inspired and taught many of the contributors.

EFFECT OF SURFACE ON THE BEHAVIOUR OF METALS. 100 pages, plates, diagrams, 5½ × 8½ in. London, Iliffe & Sons Ltd.; New York, Philosophical Library; 1958. Price, $10.00.

The four lectures delivered at the 1957 Institution of Metallurgists Refresher Course comprise the contents of this small volume. In the first paper, G. L. J. Bailey discusses modern methods of preparing and examining surfaces, and of studying their structures, both on the microscopic and atomic scales. T. P. Hoar then deals with the effects which surface treating has on the future chemical and electrochemical behavior of metals. The third paper, by F. T. Barwell, summarizes the present state of knowledge on the relationship among

surface condition, frictional resistance and wear. In the final paper, R. W. B. Stephens covers the influence of surface on the physical properties of a metal, including optical properties, skin effect, thermal properties, etc.

THE WORLD OF SCIENCE, by Jane Werner Watson. 216 pages, illustrations, 8 × 11 in. New York, Simon and Schuster, 1958. Price, $4.95.

A deluxe edition of one of the Golden Book series for young people. Beautifully illustrated with 265 color photographs and explanatory diagrams, this volume gives an exciting insight into the operation of today's modern laboratories. Encompassed are seven major fields of science: geology, astronomy, mathematics, physics, chemistry, biology and engineering. The activities of scientists rather than a survey of the history and achievements of science is the theme of this book.

KINETIC THEORY OF GASES, by R. D. Present. 280 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $7.75.

This text provides an introduction to the kinetic theory of gases for students of physics which can also be helpful to students of chemistry and the engineering sciences. Salient features include stress on limitations of the mean-free-path method; extensive use of momentum-transfer method; an evaluation of collision dynamics; introduction to Boltzmann's equation; discussion of knudsen flow and H-theorem and its consequences. The treatment of viscosity is approached at three different levels of sophistication. Each chapter ends with a selection of problems.

MAGNETIC AMPLIFIERS: THEORY AND APPLICATION, by Sidney Platt. 238 pages, illustrations, 6 × 9 in. New York, Prentice-Hall, Inc., 1958. Price, $7.00.

With a logical and thorough approach to the expanding field of magnetic amplifiers, this up to date book is designed to be especially useful to the technician and engineer. In the early chapters the fundamentals of magnetic induction and electromagnetic induction are discussed in detail. There follows a description of the theory, operation and application of saturable reactors and magnetic amplifiers in both feedback and nonfeedback cases.

ANALOG SIMULATION, by Walter J. Karplus. 434 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $10.00.

This text devoted completely to the solution of field problems by analog techniques, is divided into three major sections. Part I, Mathematical Models, deals with the techniques of solving partial differential equations by electric analogs, and also includes a treatment of finite integral transformations. Analog Systems, the second part of the book, describes the various methods of simulating field problems by analogs. Part III gives a comprehensive account of the application of the mathematical techniques of Part I and the utilization of the analog systems of Part II to the solution of engineering problems.

BIBLIOGRAFIA MARCONIANA, compiled by Giovanni di Venedetto. 243 pages, 6½ × 9½ in. Spolete (Italy), Consiglio Nazionale delle Ricerche, 1958. Price, 2000 lire (paper).

The Consiglio Nazionale delle Ricerche has published this volume as a supplement to *La Ricerca Scientifica*. The bibliography begins with a dated account of Marconi's own life—his works, the honors accorded to him—and other important dates connected with Marconi's inventions. Then follows a list of the societies to which he belonged, the honorary degrees he received, the medals he was awarded, and other honors conferred on him. The main section of the bibliography contains 2662 references, in chronological order within two groups—first, Marconi's own writings (scientific and otherwise) and second, the books and articles about Marconi. An index includes authors' names as well as subjects.

THE INTERFERENCE MICROSCOPE IN BIOLOGICAL RESEARCH, by Arthur J. Hale. 114 pages, plates, diagrams, 5½ × 8½ in. Edinburgh and London, E. & S. Livingstone Ltd., 1958. (Exclusive U. S. agents, Williams & Wilkins, Baltimore.) Price, $5.00.

Although not a new instrument, the interference microscope did not become a practical

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aid in the study of tissues and cellular processes until about a decade ago. This book describes the types of interference microscopes available (multiple beam and double beam), discusses the theoretical interpretation of the interference patterns, outlines the practical methods of measuring optical retardation, and sets forth briefly the cytochemical significance of measurements of optical retardation. The final chapter is devoted to sources of error and the appendix contains discussions of pertinent principles of physics. For those biologists who wish to learn about this relatively new technique, the book will be valuable as a self-study text.

PATTERNS OF DISCOVERY, by N. R. Hanson. 241 pages, 5½ × 8½ in. New York, Cambridge University Press, 1958. Price, $5.50.

The author, Professor of Philosophy at Indiana University, discusses the philosophical aspects of research and discovery, pointing out that the finding of theories follows a logical pattern. By means of historical examples, he shows what this logical pattern is. The first five chapters, dealing with observation, facts, causality, theories and classical particle physics, have been written with the final chapter (on elementary particle physics, or microphysics) in mind, since the author considers particle theory "the lens through which these perennial philosophical problems will be viewed." About a third of the book contains explanatory material in the form of two appendices and a voluminous set of notes.

PUNCHED CARDS, edited by Robert S. Casey, James W. Perry, Allen Kent and Madeline Berry. Second edition, 697 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1958. Price, $15.00.

In the seven years since the appearance of the first edition of this volume, so many new uses for punched cards were found that this second edition has been substantially revised by augmenting some chapters, eliminating one (on computation) and rewriting many others. Part II on practical applications is more than twice its size in the first edition, with chapters added on case histories of two commercial systems—one a Uniterm system for reports and the other a Zatocoding information retrieval system. This new edition will be welcomed not only by the users of the first edition, but also by all who are faced with the problem of trying to keep abreast of the mass scientific data being made available today.

Solar Sails.—The possibility of using huge "solar" sails to help propel space vehicles has been suggested in Detroit, Mich., by a General Electric engineer.* Norman F. Barnes, manager of information for the Company's General Engineering Laboratory at Schenectady, N. Y., said this technique could help solve one of the most serious problems in interplanetary travel—"how to provide added power boosts after the space vehicle is free from the Earth's gravitation." With conventional techniques, he said, so much energy is expended in leaving the Earth that very little remains for the rest of the trip into space.

He explained that light streaming from the sun at 186,000 miles a second could exert a force on the solar sail in much the way that wind pushes the sail of a boat.

The sail's light weight plus the free source of energy, he said, might eventually end the long search for exotic fuels to propel space vehicles through the millions of miles separating Earth from its sister planets.

If a satellite like the Pioneer were equipped with a 400 by 400-ft. aluminized, plastic sail, he said, the object could increase its speed by more than 350 miles an hour each day, and in two months of sailing would attain a speed of 25,000 miles an hour. He estimated a solar sail of this size would weigh less than 100 pounds. Development of the solar sail technique, however, is still too far in the future to solve immediate problems of space probing, he said.

The technique, he said, would eliminate need for heavy, bulky, more conventional fuels.

He explained that particles of light, called "photons," are continually streaming from the sun at the highest speed known to man. "When these particles of light fall upon a material and are absorbed by it, the light exerts a pressure on that substance called 'radiation pressure.'"

Very thin plastic sheet similar to that now commercially available, he said, could be used for the sail. Aluminizing one side of it would permit the sunlight to be reflected as by a mirror —"doubling the radiation pressure."

Barnes said the sail could be completely folded within the space vehicle during launching from the Earth, then opened in space.

"Since there would be no air resistance," he said, the "unfurling would not be difficult. Very large sails could be used. Shroud lines would transmit the pull back to the ship. Though the radiation pressure of sunlight is very slight, even this small force on the sail would be enough to produce a rather large increase in speed."

For a 25-lb. satellite, he said, a 200-by-200-foot sail would produce an increase in speed of more than 300 miles an hour each day. "In other words, at noon one day the satellite would be going 300 miles an hour faster than at noon the day before. In 10 days, the speed could be increased by 3,000 miles an hour."

British Telescope Followed "Pioneer."—Britain's giant radio telescope

* EDITOR'S NOTE: This idea was previously expressed by Dr. I. M. Levitt during The Institute's "Ten Steps into Space" lecture series during the spring of 1958, published as JOURNAL Monograph No. 6, in December, 1958.

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at Jodrell Bank in Cheshire, England —the largest steerable radio telescope in the free world—received priceless information from the American moon rocket "Pioneer" fired from Cape Canaveral on October 11 by the U. S. Air Force.

Professor A. C. B. Lovell, director of Jodrell Bank radio-astronomy station, reported that "for the first time we have had two full days of information about conditions in space up to 80,000 miles—regions which no man-made objects have ever penetrated before."

From Jodrell Bank to America went no fewer than 112 "fixes" of the rocket's positions—all of them accurate to within one-fifth of a degree. Professor Lovell said they had never had such powerful signals before. Signals from the rocket, first picked up by Jodrell Bank ten minutes after firing, were very strong from the start.

"Here we have been thrilled to join in this experiment, ... Britain is proud to have cooperated. So far as the telescope is concerned, it has been 'a piece of cake' (R.A.F. language for 'like falling off a log'). If the next one is as easy to contact I shall be delighted," said the Professor.

Professor Lovell added that with the exception of determining the magnetic field and getting a picture of the back of the moon, all experiments had been 100 per cent successful.

Radio information giving instrument readings of temperatures, meteorite impacts, and ion content in space was received by the radio telescope in the form of coded signals. There was no interference and Jodrell Bank could see quite clearly on a chart the rotation of the rocket—about 110 revolutions a minute. These signals were piped from the collecting aerial at the focus of the telescope bowl to a U.S. Air Force mobile equipment van manned by U. S. scientists, near the control building. All the information from the rocket was relayed from the telescope via two telephones and a teletype line in the trailer through to America.

Professor Lovell spoke of the success achieved by the radio apparatus worked by the American team. This in conjunction with the telescope had, he said, given "a handsome measure of fixes." If the rocket had had the energy to go into the regions beyond the moon, he said he was quite certain Jodrell Bank would have had no difficulty in tracking it provided the batteries had continued to operate.

Mr. W. D. Young of the American team paid tribute to "a wonderful crew" at Jodrell Bank where Lovell's men had not only cooperated in a great many ways but had gone "above and beyond the absolute requirements."

The Jodrell Bank telescope was completed in 1957 to the order of Manchester University jointly with the U.K. Department of Scientific and Industrial Research. It takes the form of a great bowl-shaped reflecting surface, 250 ft. in diameter and weighing about 800 tons. With this instrument, signals can be transmitted to the moon and back in two and a half seconds or to Venus and back in four minutes.

New Ammonium Perchlorate Plant. —Four million pounds per year of ammonium perchlorate is the approximate initial capacity of the plant now being built by HEF, Inc. near Columbus, Miss., according to R. Wolcott Hooker, President. Ammonium perchlorate is an important oxidizer now in demand for use in high energy solid fuels for guided missile propulsion. With construction well under way and most of the equipment already delivered, Mr. Hooker expects full-scale production in February.

Pilot plant operation of the entirely new, jointly-developed process proves that either ammonium or lithium perchlorate can be produced with equal facility, should emphasis be switched to lithium perchlorate in the missile program. The HEF plant (owned jointly by Hooker Chemical Co. and Foote Mineral Co.) is the nation's only plant capable of producing lithium perchlorate commercially and the dual function is considered of basic importance. Missiles research indicates a very important potential for the lithium compound in this fast-expanding field where new technology is continually being developed.

Spherical Kaolin Catalyst.—Kao-Spheres, a new spherical Houdry Mineral Kaolin cracking catalyst, designed to decrease catalyst use and reduce equipment maintenance costs, is being offered by Houdry Process Corporation, of Philadelphia. The Kaolin catalyst itself provides maximum octane at low cost, high activity, sulphur resistance, high thermal stability and excellent regenerability. The spherical form sharply reduces catalyst chipping and loss in fines. Less maintenance is needed because the Kao-Spheres do not have sharp edges that may cause equipment erosion. The new spherical form is expected to provide refiners, especially catalytic crackers, with a more economical method of operation while retaining all the advantages of the high-octane, low-cost Houdry Mineral Kaolin. The catalyst will be especially effective for Houdriflow, and all other moving bed catalytic cracking units.

Subminiature Emitter-Follower.—A Subminiature Emitter-Follower, the Model 5000, designed for missile, satellite and airborne requirements was introduced recently by Columbia Research Laboratories, Woodlyn, Pa. This completely solid state instrument combines significant savings in space and weight with superior performance characteristics for impedance matching of piezoelectric transducers in telemetering applications.

The unit measures $1\frac{1}{8} \times 1\frac{1}{4} \times \frac{5}{8}$ in. and is potted in an aluminum case. It will operate in ambient temperatures between $-65°$ F. and $+185°$ F. under high noise and shock conditions. Input impedance of 70 megohms permits measurements from 5 cps. to 100 kc. with crystal type accelerometers, and the output noise characteristics are such that signals in the low millivolt region may be handled. Output is linear over a 20-volt input range and the circuit draws approximately 660 μa from a 28 vdc. supply. It provides a current gain of 50,000 and a voltage gain of 0.99. Other models are available with a continuously variable voltage gain to 100×. The basic circuit of the unit consists of three emitter follower stages cascaded to attain a high input impedance. There is a-c. coupling between stages, and all internal power supplies have been eliminated by the use of Zener diodes.

Television for Stratoscope Balloon Flights.—Television will join a telescope in balloon flights 80,000 ft. above the Earth, as part of a project supported jointly by the National Science Foundation and the Office of Naval Research. A 36-in. telescope, Stratoscope II, will be constructed for use in conjunction with further flights of the 12-in. Stratoscope I telescope. Both systems will be equipped with a television link, with control of the telescope from the ground, for remote-control pointing of the telescopes at celestial objects. The project will be directed by Dr. Martin Schwarzschild of Princeton University's Department

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of Astronomy, who also directed the Stratoscope I flights in 1957.

The 1957 flights, made under the sponsorship of the Office of Naval Research, resulted in the first successful attempts at high altitude astronomy. The special 12-in. sun telescope camera made two unmanned balloon flights and took photographs of the sun's surface which were unprecedented in clarity and detail. The new program is in essence a continuation of the earlier ONR-backed effort.

Purpose of sending telescopes up in balloons is to get the instruments above the Earth's atmosphere, which obscures clear visibility of heavenly bodies. Telescopes, no matter how large, which are at ground level can virtually never give better definition of objects at night than that given by a 12-in. instrument, because of this atmospheric obstruction.

The first flights to be made in the new program will probably occur during the summer of 1959 with Stratoscope I. The much larger Stratoscope II is now the subject of a design study by the Perkin-Elmer Corporation, Norwalk, Conn., which designed and built Stratoscope I. Design work will probably require the remainder of 1959, with construction to take place in 1960. After tests, Stratoscope II's first flight is scheduled to take place in the summer of 1961.

The Stratoscope I instrument will continue to be used for solar research. Stratoscope II will be used for high-resolution photography of objects other than the sun, such as the planets, galaxies, and nebulae.

Dr. Schwarzschild's work has been acclaimed by astronomers as among the most significant scientific advances in the last decade. Stratoscope I's photographs of the sun showed clearly for the first time the detailed polygonal structure of the convective currents which bring much of the internal solar energy to the surface of the sun. Dr. Schwarzschild and his co-workers were awarded the Newcomb Cleveland Prize of the American Association for the Advancement of Science in December 1957 for their outstanding contributions made as a result of Stratoscope I.

Total Synthesis of Yohimbine.—Solution of one of organic chemistry's classic problems, total synthesis of yohimbine, has been reported by chemists at the University of Wisconsin led by Prof. Eugene van Tamelen. Yohimbine is an alkaloid or organic base—a compound containing nitrogen, carbon, hydrogen and oxygen. In the Congo and various other parts of the world it is found in nature as a component of the bark of certain trees. Morphine, cocaine and strychnine are more commonly known alkaloids.

Yohimbine was first isolated as a pure substance in the 1890's. Since then organic chemists have tackled the synthesis problem without success until the breakthrough by van Tamelen and collaborators Maurice Shamma, Albert Burgstahler, Joseph Wolinsky, Rudolph Tamm, and Paul Aldrich.

Attempts at chemical synthesis are premised on exact knowledge of the chemical structure of a naturally occurring compound. By 1945 yohimbine was pretty well defined structurally. Van Tamelen pinpointed it further in the early 1950's, and the synthesis task began in earnest in 1954.

A series of 20 steps was required, each one contingent on the successful completion of a prior operation. Van Tamelen characterizes the four-year research effort as "a prodigious amount of work."

Chief among many complicating factors is the order of reduction of the yield after successive processes. Be-

ginning with pounds of starting materials, the unit of quantitative measurement shrinks to milligrams in the latter stages.

Butadiene, a by-product of gasoline manufacture, and quinone, a coal tar product, were the starting materials. Careful control of variables such as temperature, reaction time and solvent, was necessary to insure success of each individual step. After re-runs at virtually every stage in the process, the project was completed and a preliminary report published in the *Journal of the American Chemical Society*, Sept. 20.

Van Tamelen likens organic synthesis work to mountain climbing. Each step is a starting point for the next one. He pushes the comparison still further to explain the "why" of a synthesis effort. "It's undertaken simply because the natural compound is there," he says.

The synthesis of yohimbine is significant because it is a member of a family of alkaloid materials which are physiologically active. Reserpine, the drug used in lowering high blood pressure and famous as one of the first tranquilizers, is very similar chemically to yohimbine. But its effect on the body is vastly different. Yohimbine is an aphrodisiac; that is, it stimulates the physical, sexual centers of the body. This factor has caused it to be used extensively for breeding purposes in veterinary medicine. A far less technical source of interest is the fact that it is the basis of the tales of a "love potion," originating in Africa.

New Tension Meter.—A new instrument with a range as high as 50 lb. checks tension on moving yarns, wires, sheets, tapes, and other filamentous and also flat materials.

The new tension meter, made by Tensitron, Inc., Harvard, Mass., features a novel lever inserting mechanism. Both outer rollers are lowered simultaneously, which permits placing the wire, cord or tape between the rollers. Upon moving of the lever in the other direction, the wire is put in test position. Now the sensing roller in the center is lifted elastically in proportion to the strain. This motion, amplified over a shockless spiral movement, is indicated by the dial with good legibility. The gage mechanism averages the readings for steady observation of the average tension though the latter be fluctuating.

The housing is finished in black wrinkle giving a sure grip even with wet hands. Protruding parts on the aluminum base plate protect the fingers from contact with cutting wires or cords at high speed. Precision ball bearings in the rollers permit the use of the Tension Meter even when the wires travel at thousands of feet per minute.

The new tension meter can be inserted and readings taken while the wires or other materials are running. No stopping of the machine is necessary to make a tension reading.

This modern tension meter of advanced design can also be made with flat 1-in. rollers, to be inserted into the side of a moving sheet of cloth, plastic or paper. Simple multiplication with the width of the sheet in inches indicates then the total tension.

Liquid Flywheel to Control Space Craft.—A "liquid flywheel" for controlling space craft has been invented by Robert P. Haviland, space engineer with General Electric's Missile and Space Vehicle Department. His patent No. 2, 856, 142 is for a device to keep a vehicle properly oriented during its flight through space.

The apparatus, which Mr. Haviland calls a "liquid flywheel," uses a hoop-like arrangement of pipes—the size of the hoops are limited only by the size of the space craft—through which is pumped a liquid at varying speeds. The change in the liquid determines the amount of force applied in any one direction.

Such a control device is needed, according to Mr. Haviland, because the nearly perfect vacuum conditions of outer space will mitigate against using fins, rudders, and other devices dependent on air drag.

"This pipe arrangement," said Mr. Haviland, who is world-famous for his long career in the missile and satellite field, "will provide the added advantage of storing needed liquids—water, for example—in the hoops, and then using this stored liquid as the force medium."

New Developments in Thermocompression Bonding.—Recent developments in thermocompression bonding have made it possible to form large area contacts between metallic leads and semiconductor devices, according to H. L. Mellor of Bell Telephone Laboratories. Use of the technique to form contacts as much as 30 mils in diameter was described in a paper presented at the 1958 Conference of the IRE Professional Group on Electron Devices.

It is also now possible to attach leads to opposite sides of a semiconductor wafer simultaneously, thus making both base and emitter contacts at once.

In thermocompression bonding, soldering and alloying techniques are eliminated, along with undesirable low melting materials and fluxes. In its simplest form, a soft metal lead is pressed against the gold-plated surface of a semiconductor, and moderate heat applied. Adhesion generally takes place in a few seconds. Pressures in the range of 12,000 to 20,000 psi. are normally used; temperatures required are in the 300° to 325° C. range. These temperatures are well below the melting points of either the lead or the semiconductor, and the pressures are below the deformation point of the semiconductor.

For large area contacts, gold or silver leads from 6 to 15 mils in diameter are "headed" like nails. Heads are commonly 30 mils in diameter. These headed leads are fitted into a hollow jig in a ram, and pressed against the surface of the heated semiconductor. Wires with flattened ends can also be used in this manner, when the diameter of the lead is too small to make heading practical.

Thermocompression bonding in this manner is being used to attach leads to diffused silicon diodes at Bell Laboratories. The 30 mil contact head covers almost the entire active area of the mesa-type diode. This configuration allows the passage of a high forward current.

Specifications for the use of thermocompression bonding in the commercial preparation of diodes and transistors are being prepared by Bell engineers. The method will allow the use of higher bake-out temperatures, and is an inherently cleaner procedure than present methods of attachment. Both considerations result in greater reliability of finished semiconductor devices.

Cryopumping.—An unconventional technique for producing extremely high vacuums promises substantial savings for research and industrial uses. It is already employed for missile research in a Universtiy of South-

ern California wind tunnel. The technique, called cryopumping, was explained to members of the American Vacuum Society attending the Third National Vacuum Symposium in San Francisco.

Cryopumping produces such a low temperature that all of the air in a chamber or wind tunnel freezes to a solid, leaving a high vacuum. Details of the technique were given by Bruce M. Bailey, of Arthur D. Little, Inc., research and engineering company of Cambridge, in a paper co-authored with Dr. Raymond L. Chuan, Director of the University of Southern California's Engineering Center. Dr. Chuan developed the first practical, large-scale application of the technique—a hypersonic wind tunnel used to test missile models under high altitude atmospheric conditions. Bailey and his colleagues at ADL developed the cryopump used to freeze air out one end of the tunnel, providing high-speed air flow over the missile models. The wind tunnel was unveiled in October, 1958.

The cryopump is an extreme low temperature refrigerator, using helium gas at minus 420° F. as the refrigerant. At present cryopumping is most efficiently used in conjunction with mechanical pumps. Whereas the pumping speed of a mechanical system falls off rapidly as the vacuum increases, a cryopump works particularly well when a relatively high vacuum exists. Such a combined system makes use of the advantages of both methods.

With the cryopump, it is now possible to produce extremely low pressures (about one one-millionth of our normal atmospheric pressure) economically enough for large scale industrial applications. The temperatures involved are those believed to exist on the dark side of the moon.

Description of Hypersonic Wind Tunnel

Instead of blowing or pulling air over the test models by conventional means, which would require pumps and fans filling almost an entire city block, the cryopump freezes nitrogen gas out of the stream, causing a flow of nitrogen from the other end of the chamber and over the objects being tested. Nitrogen is used instead of air to eliminate the need to remove moisture from the gas stream.

The temperature of the tunnel at this point is around −450° F. In order to duplicate high temperature conditions under which missiles would operate at high speeds, the nitrogen gas is heated by electricity when it first enters the tunnel and again just before it flows over the models. After the nitrogen passes over the models, it is drawn onto the large metal plates of the refrigerator, where it condenses and freezes. Missiles placed in the tunnel can thus be tested as though they were flying at 20 times the speed of sound and nearly 60 miles (300,000 ft.) above the Earth.

The USC tunnel is unique in that its only moving parts are in the refrigerator, which runs on a 50-hp. motor. A comparable unit based on a gaseous diffusion pump would require 500,000 hp. The tunnel can run 10 hr. at a stretch, but most hypersonic test facilities can operate only a fraction of a second at top speed and altitude.

The tunnel was built on a research contract for the Air Force Office of Scientific Research and the Office of Naval Research. Arthur D. Little, Inc. designed the refrigeration system.

Satellite Observatories.—Satellite observatories for getting an unobstructed, 24-hr. a day look at the sun may be just around the scientific

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corner, according to Prof. Leo Goldberg, Chairman of the Department of Astronomy at the University of Michigan.

Such a solar satellite has already been designed by a St. Louis Aircraft Manufacturer, Professor Goldberg said.

Professor Goldberg made his prediction while delivering the 25th Annual James Arthur Lecture on the Sun at the Smithsonian Institution in Washington.

The biggest problem of solar observation has been obstruction by the Earth's atmosphere of ultra-violet radiation emitted from the sun—important to the study of the sun's characteristics. Visible light, which is what observers on the ground use as a basis for their study of the sun, doesn't tell the whole story.

The satellite observatory, scheduled for intensive laboratory experimentation, would circle 400 miles above the Earth and thus interference by the atmosphere should be completely eliminated, Goldberg said.

Describing the satellite, he revealed: "The entire instrument package will weigh less than 300 pounds and will include a stabilization and control system for precision pointing of instruments at the sun, power supply, and equipment for recording data and transmitting the information upon command from Earth."

Professor Goldberg cautioned, however: "Much in the way of laboratory experimentation will be required to ensure that the satellite and instrumantation will withstand the severe accelerations and vibrations during launching operation, and also to make sure the systems envisaged for the stabilization and pointing control system and for the recording, storage and transmission of data function correctly."

Goldberg added that the experimental scheme is based on satellite payloads "that will quite certainly be feasible within the coming year or two."

Up to now, he said, the greatest limitation of rockets in solar research has been the short space of time they have had to record data. Placing a large rocket in a stable orbit high above Earth, which now looks possible, will give the astronomer an "unobstructed view of the universe," he said.

American astronomers were far from shocked by Russia's early lead in the field of satellites, the U-M scientist said. "However, even our scientists were taken aback by the enormous sizes of the Russian Sputniks.

"The initial shock has now worn off and has been replaced by eager anticipation of the consequences for astronomy of artificial satellites that can carry several thousand pounds of scientific equipment into orbits hundreds of miles above the surface of the Earth."

Our sun—"the most important star in the sky"—"should have the highest priority for investigation from satellites," Goldberg said, because:

1. The sun is such an intense source of radiation that instrumental problems are at a minimum;
2. The influence of the sun upon Earth is of "very great interest to geophysicists, and meteorologists as well as astronomers;
3. Observations from the ground during the past have clearly defined goals of satellite research.

"Solar activity initiates various events on the Earth, chiefly in the upper atmosphere, and hence its study has very important practical consequences for the human race," he said.

"To comprehend the physical nature of (the sun) and to understand the causes of disturbances to the Earth's

atmosphere, it is necessary to observe directly the ultraviolet radiation that is now screened from observation by the Earth's atmosphere," Goldberg declared.

Motion Pictures of Dislocations in Graphite.—Dislocations, or imperfections, in the atomic arrangement of the structure of graphite, an important material used in nuclear reactors, have been recorded for the first time by motion pictures at Atomics International, a division of North American Aviation, Inc.

Dislocations affect the physical properties of materials, influencing the strength, conductivity and other mechanical properties. The discovery has established a basis for continued studies, which may indicate new practical applications of the material. It is being documented by additional research.

Graphite is used by Atomics International as a moderator in several nuclear reactors, including the Sodium Reactor Experiment (SRE), an atomic power project conducted by the company for the Atomic Energy Commission. The SRE has produced heat for the generation of nearly 2,600,000 kilowatt hours of electricity for commerical use in Southern California during several power runs.

Observation of dislocations in graphite were disclosed by Dr. Alexander Grenall, research specialist at Atomics International, at a recent meeting of the Electron Microscope Society of America at Santa Monica, Calif. Dislocations and their nature were revealed by an electron microscope.

Latest photographic equipment was used to record the movements of dislocations in graphite crystals. A 16-millimeter motion picture camera was mounted so that the lens was near the microscope's fluorescent screen. A special lens of f/0.95 and high speed film made possible the film recording.

Graphite specimens were photographed at magnifications of 12,500 and 25,000. Only the light from the microscope screen was utilized in the procedure.

Dr. Grenall classified dislocation motions as rapid or slow. Rapid movements were jerky and completed their action almost instantly, while slow motions involved gradual drifting of dislocations.

The movement of dislocations appeared to differ depending whether they were associated with a single dislocation or a network of imperfections, the scientist concluded.

"Messenger" from Outer Space.—The Air Force has officially disclosed that it has been recovering recorded scientific information from outer space with some of its Thor and Atlas missile flights that carry the operational nose cone. According to the Air Force, the General Electric-developed nose cone, which can be carried by either the Atlas or the Thor, is equipped with a "messenger" that records critical scientific data during a missile's flight. The canned information is thrown free, before the cone strikes the Earth, to fall independently.

The "messenger," called a data recovery capsule by its GE developers, is built as a plastic sphere 18 in. through. The plastic is strong enough to protect the capsule's recording instruments, yet light enough to allow the capsule to float if it falls into the water.

To date, all Atlas and Thor flights have been over water, and all the capsules used on Thor flights have been recovered. (One Atlas capsule floated for over two months and was found by some Barbidian fishermen, who called it a "Russian Sputnik." The infor-

mation on the tape was still intact, and was scientifically useful.)

After ejection from the nose cone, at some predetermined point before impact, the "space messenger" falls free. Upon falling into the sea, its locating devices are powered by a battery that is triggered by sea water. Included in the capsule are a sofar bomb and a radio beacon, to allow for triangulation search techniques, and dye markers, to permit visual search. To protect the valuable, plastic-encapsulated instruments from sharks, a shark-repellent is included in the dye.

In its flight-test program for nose cone development, the Air Force relies on radio transmission of encoded flight data back to ground stations. If, for any reason, the transmission of this information is interrupted, their "messenger" will substitute tape-recorded data. The tape recorder in the data capsule receives and stores, during its recording periods, all data telemetered from the nose cone during flight. Such data include temperature, pressure, stress, acceleration, and deceleration information.

"By this method of data-preservation," say Air Force officials, "recovery of the nose cone itself is not necessary, for its recovery can add nothing significant to information already received through telemetered and recorded data."

The General Electric Company's Missile and Space Vehicle Department, developers of the nose cone and its "space messenger," acknowledged, at the same time, the help of several of the nation's business firms who contributed to the capsule's development. According to GE, Ampex made the tape recorder, Allied Chemical supplied the Dupont-produced urethane foam, Standard Plastics molded the parts of the sphere, and many other firms contributed components.

"Without the diverse talents of the Air Force's industrial weapon systems developers," said GE, "such unique contributions as the data capsule might take years to evolve. Basic research supported by industrial laboratories has meant the difference between developing readily available weapon systems and the lag time involved in starting from scratch."

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